Quarkonium on the Lattice

Gunnar Bali (Regensburg)

New states

Spectroscopy: $n_f = 0$ and $n_f > 0$

Radiative transitions

Quarkonium decay and mixing

Summary

ECT★ Trento, 4.7.6

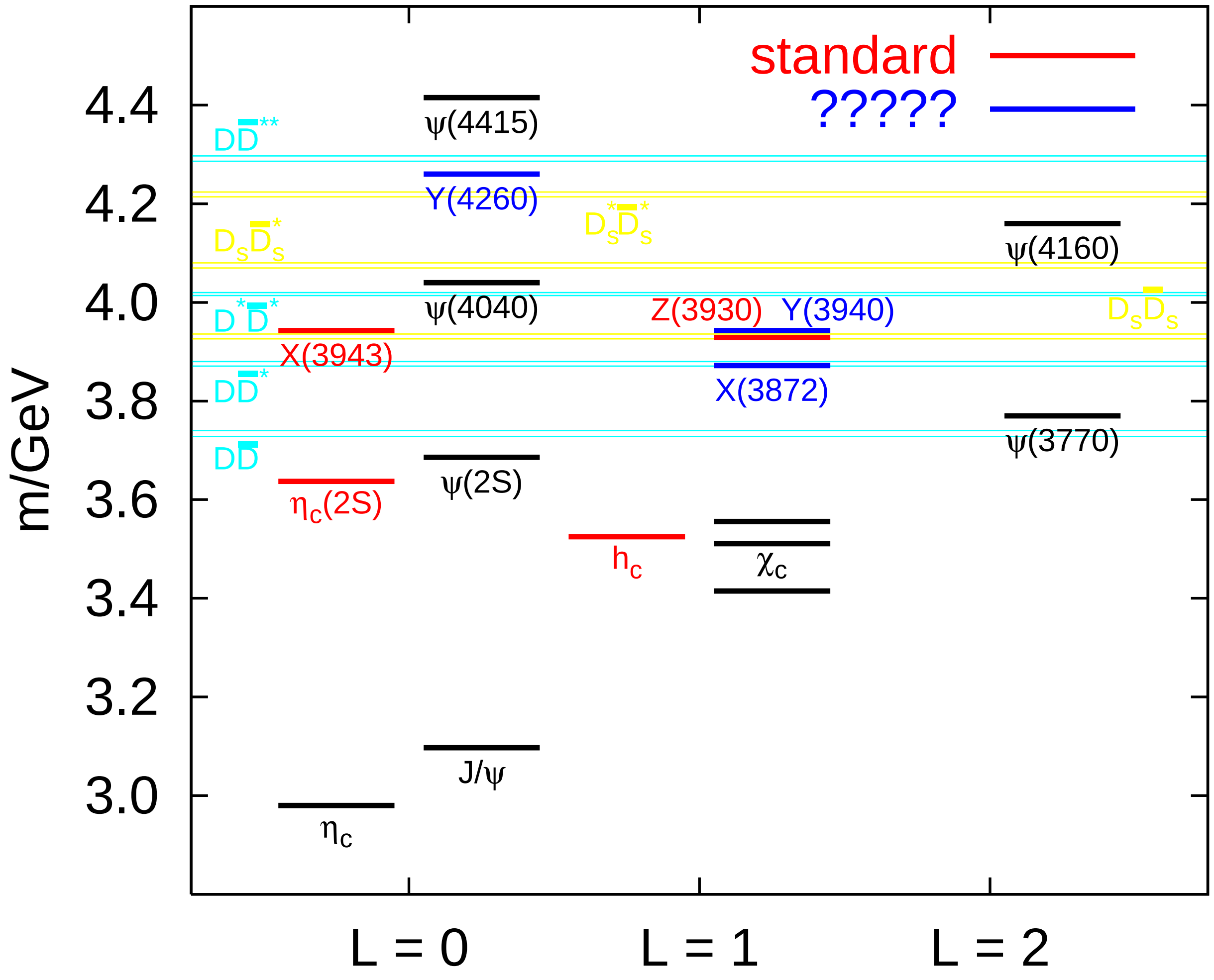
Spectroscopy: exciting times

First time in > 20 years: several new narrow < 10 MeV resonances!

- ★ Υ D wave(s)
- ★ B_c
- ★ η_c', h_c
- ★ $X(3872), X(3943)$
- ★ $D_{sJ}^*(2317)$, $D_{sJ}^*(2460)$, more D_{sJ}^*'s ???
- ★ Baryonic news

1974 – 1977: 10 $c\bar{c}$ resonances, 1978 – 2001: 0 $c\bar{c}$'s

2002 – 2005: 7 new $c\bar{c}$'s discovered by BaBar, Belle, CLEO-c, CDF, D0



new detectors

higher luminosity

new channels:

B decays

$\gamma\gamma$

$\psi\psi$-production

gg in $p\bar{p}$ collisions.

$c\bar{c}g$ hybrids ?

$c\bar{q}q\bar{c}$ in $c\bar{c}$?

www. Googlism .com:

Gluonic excitations: the necessary first step in understanding the nature of confinement.

determined from first principles using lattice QCD.

Gluon: a registered trademark of gluon.

working with a variety of innovative companies to deploy revenue.

secured to the wall with a lock.

precisely as sticky as our theory predicts.

Exotica: a strip club in Toronto.

Hybrids: not illegal in Sonoma county.

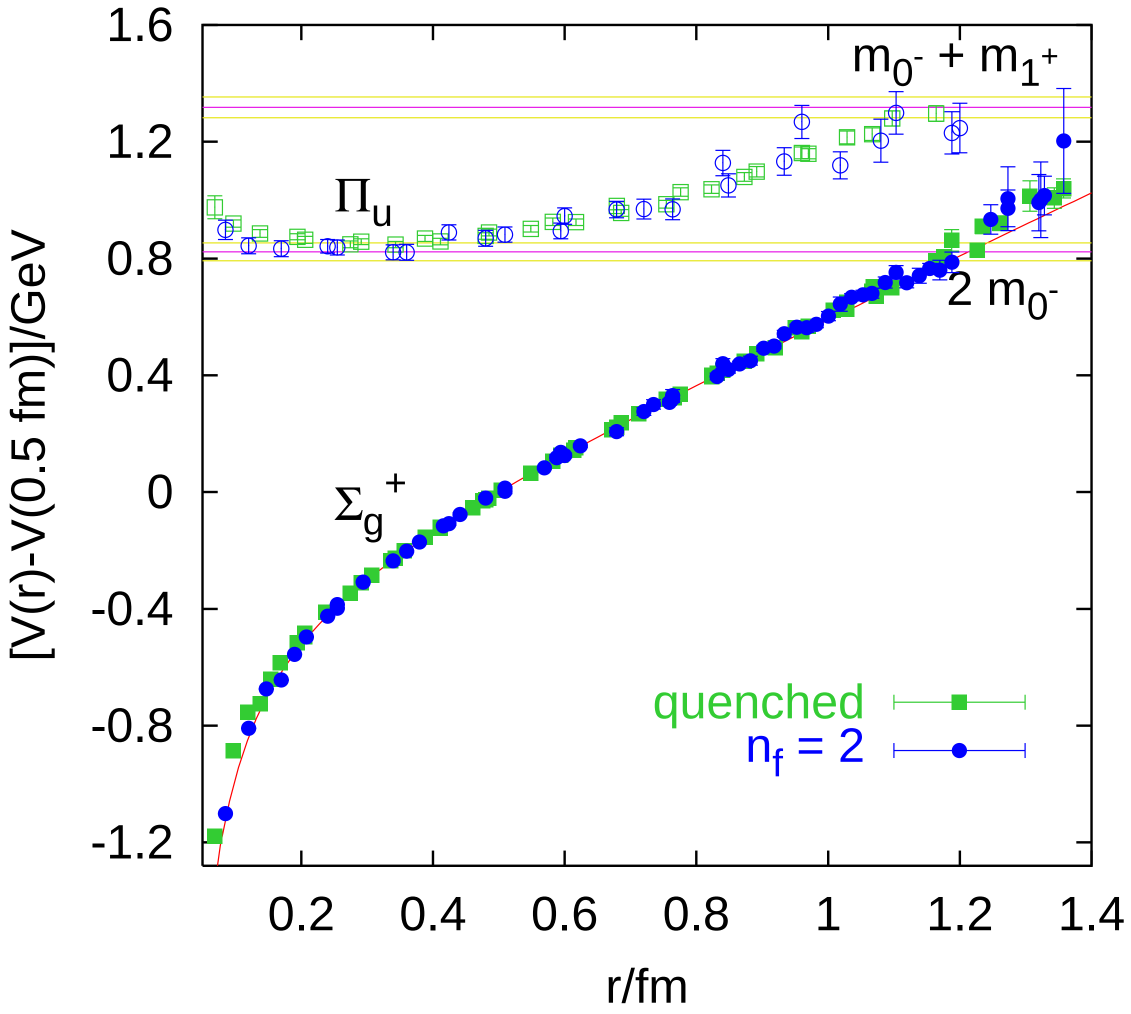
lucky enough to have it both.

Hybrid mesons

Heavy quarks:

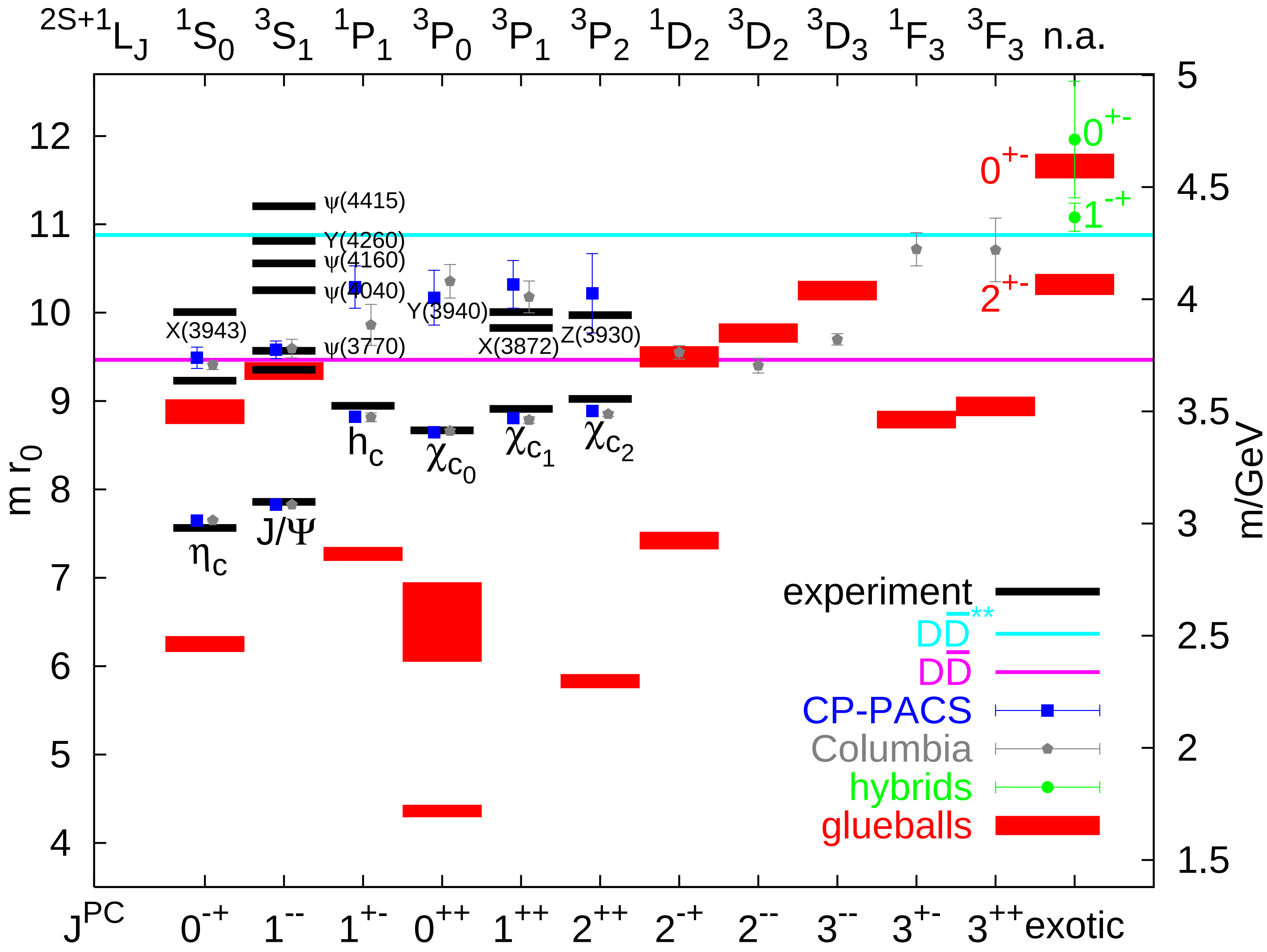
$$H\psi_{nlm} = E_{nl}\psi_{nlm}$$

$$H = 2m_Q + \frac{p^2}{m_Q} + V(r)$$



Quenched Lattice: glueballs, charmonia and hybrids

(No "disconnected diagramms" and no sea quarks $\longrightarrow$ no mixing $G, c\bar{c}, c\bar{q}q\bar{c}$, no decay !)



Columbia: 1^{--} $cg\bar{c}$ decays fast into J/ψ.

T. Burch et al 03: hybrid content of $\Upsilon(1S)$ tiny.

Liu & Luo 05: no hybrid in ψ' but a lot in J/ψ ????

Columbia: vector $m(^3D_1) \longrightarrow m(1^3S_1)$.

quark mass: $m_c \approx 1.3$ GeV

binding energies: $\bar{\Lambda} \approx 400$ MeV

$La \gg \bar{\Lambda}^{-1}, \quad a \ll m_c^{-1} \quad \Longrightarrow \quad L > 20$

one possibility: anisotropy, $a_t < a_s \quad \longrightarrow \quad L_s < L_t$

$a_s > a_t$:

Columbia: X. Liao, T. Manke, hep-lat/0210030; P. Chen, PRD64 (01) 034509.

CP-PACS: M. Okamoto et al, PRD65 (02) 094508.

Z.-H. Mei, X.-Q. Luo, IJMP A18 (03) 5713; Y. Liu, X.-Q. Luo, PRD73 (06) 054510.

J. Dudek, R. Edwards, D. Richards, PRD73 (06) 074507 (domain wall).

$a_s = a_t$:

MILC: C. Bernard et al, PRD56 (97) 7039.

P. Boyle, hep-lat/9903017.

QCD-Taro: S. Choe et al, JHEP 0308 (03) 022; P. de Forcrand et al, JHEP 0408 (04) 004.

UKQCD: C. McNeile et al, PRD70 (04) 034506.

χQCD: S. Tamhankar et al, PLB638 (06) 55 (overlap).

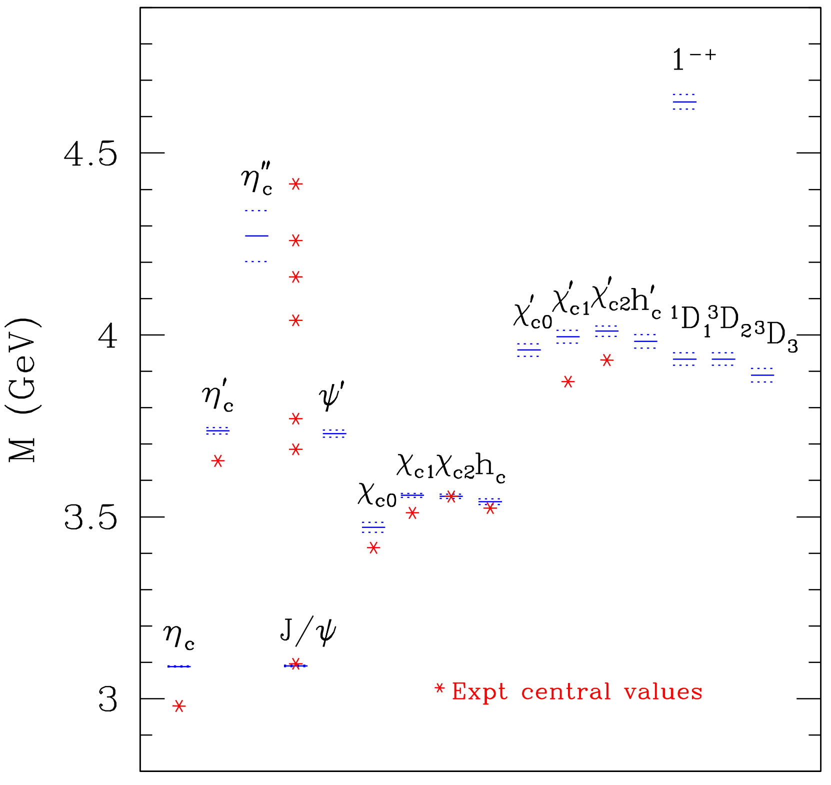
with sea quarks:

TrinLat: J. Juge et al, PoS LAT2005 (06) 029 ($n_f = 2, a_s \approx 6a_t$).

FNAL+MILC: S. Gottlieb et al, PoS LAT2005 (06) 203 ($n_f \stackrel{?}{=} 2+1, a_s = a_t$).

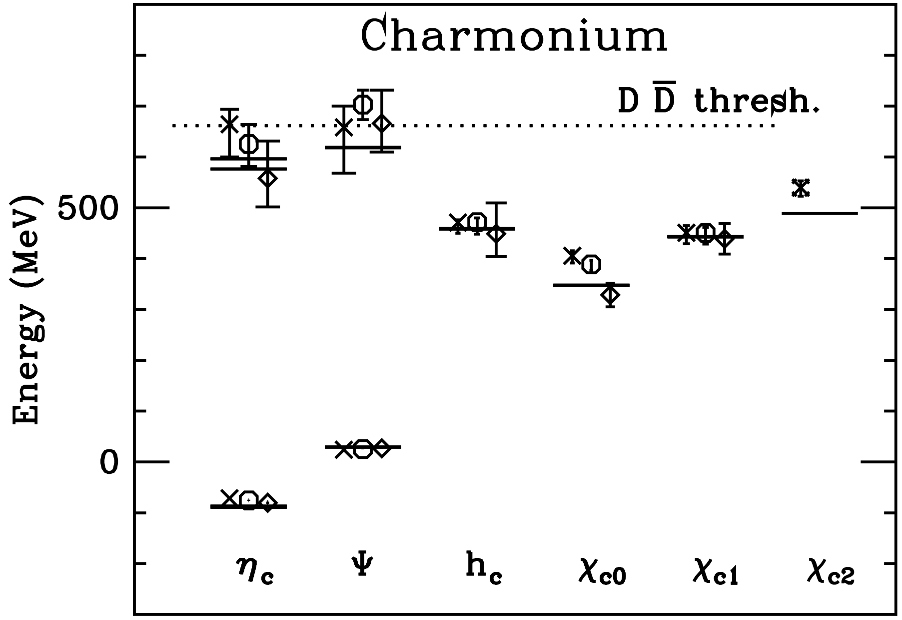
First results with sea quarks

TrinLat (preliminary: $L_s = 8$!)



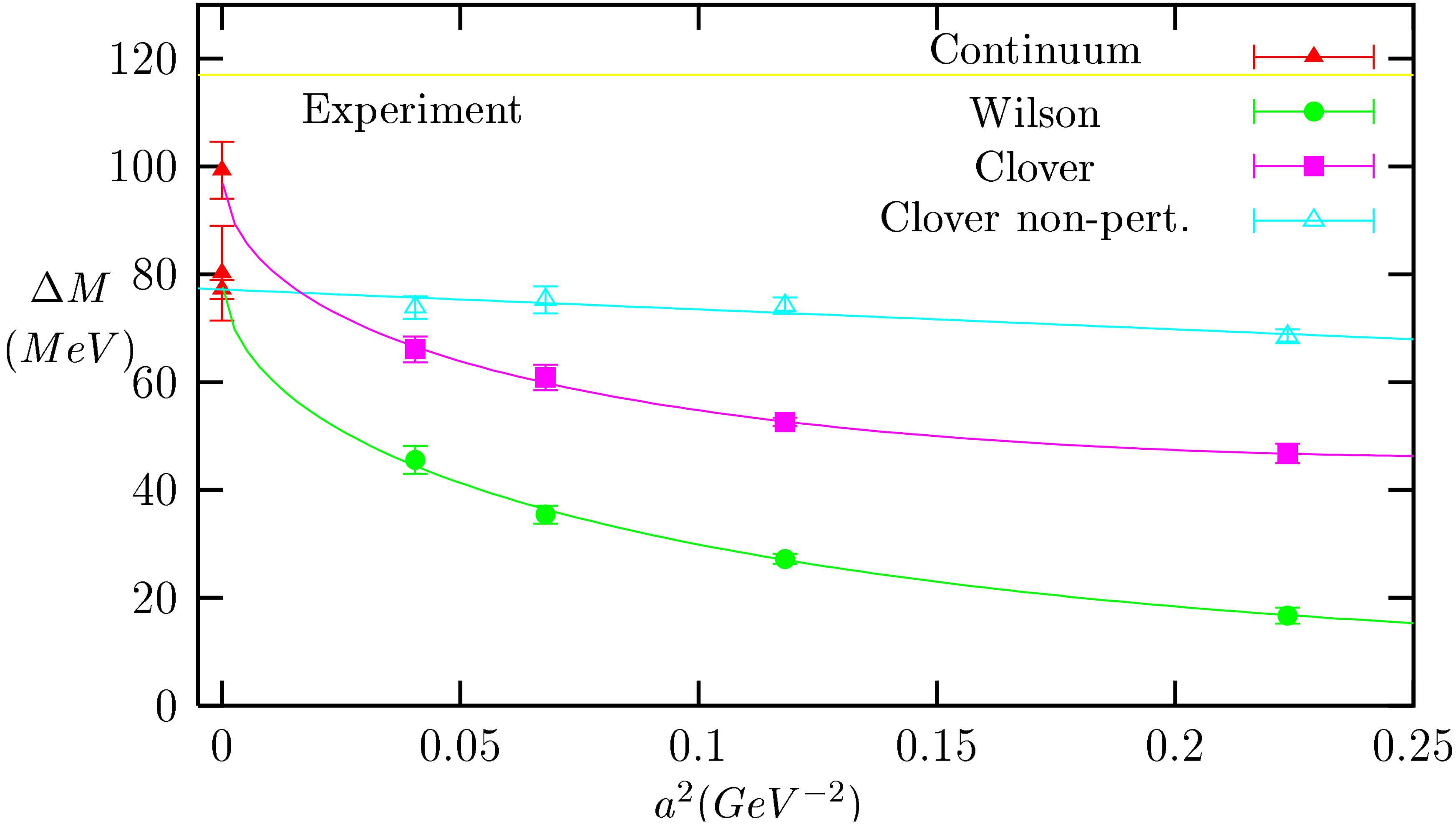
$a_s^{-1} \approx 1.3$ GeV

FNAL+MILC ($n_f \stackrel{?}{=} 2+1$)



$a^{-1} \approx 1.1, 1.6, 2.3$ GeV.

QCD-TARO: quenched $\Delta M = m_{J/\psi} - m_{\eta_c}$



NRQCD: $\Delta M = \frac{1}{6m_c^2}\langle\psi|V_4|\psi\rangle + \cdots$

Leading order perturbation theory: $V_4(r) = 8\pi C_F \alpha_s \delta^3(r)$.

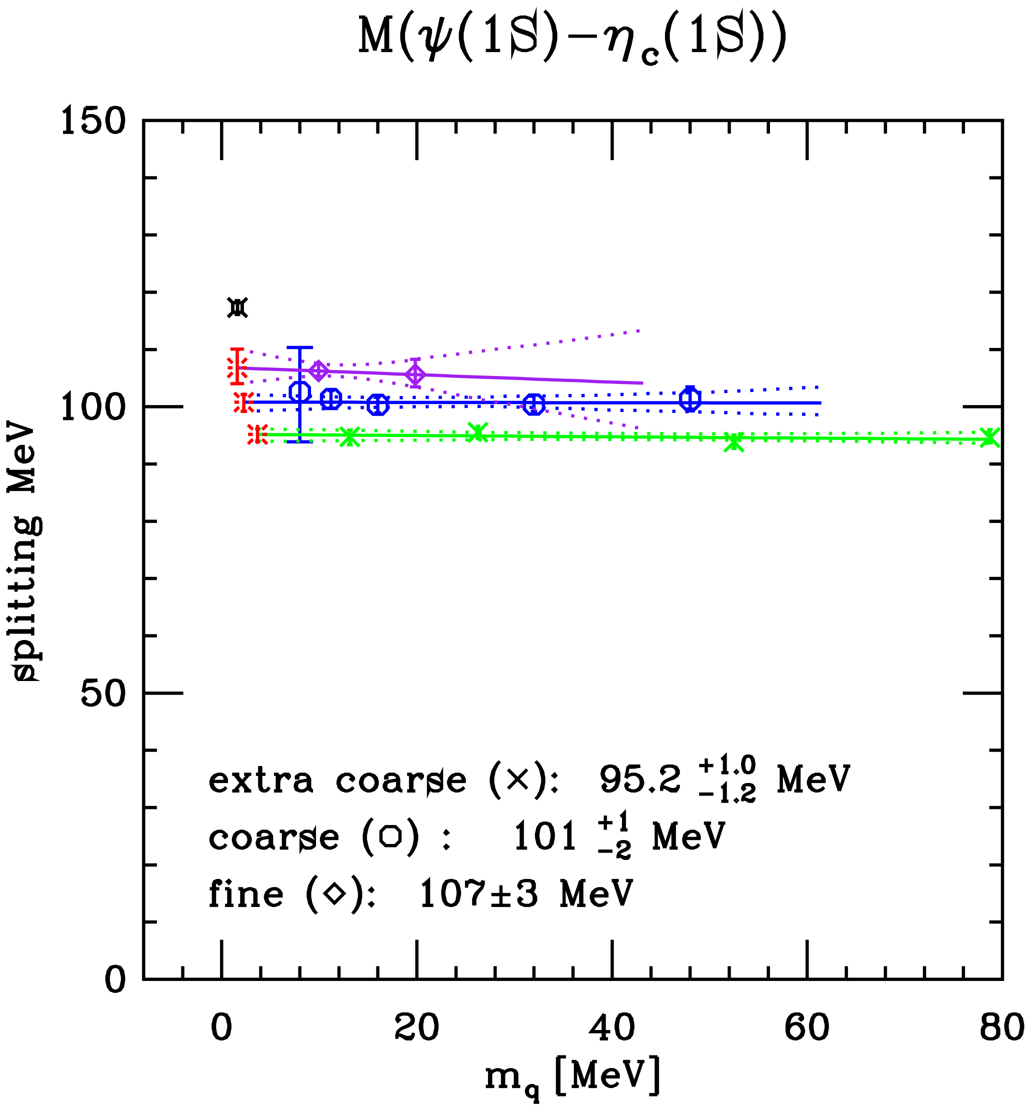
ΔM	scale from r_0	scale from $1\overline{P} - 1\overline{S}$
Columbia	72(2) MeV	83(??) MeV
CP-PACS	73(1)(4) MeV	85(4)(6) MeV
QCD-TARO	77(2)(6) MeV	89(??) MeV
χQCD	88(4) MeV	121(6) MeV
JLAB	97(6) MeV	???

JLAB (Dudek et al): $m_c \approx 5\,\%$ too small !

χQCD (Tamhankar et al) $+$ JLAB: only one lattice spacing a.

χQCD: $La < 0.9$ fm $\rightarrow$ $1\overline{P} - 1\overline{S}$ underestimated ?

FNAL+MILC: $n_f \stackrel{?}{=} 2+1$



$a^{-1} \approx 1.1, 1.6, 2.3$ GeV.

$\Delta M \longrightarrow$ 117 MeV

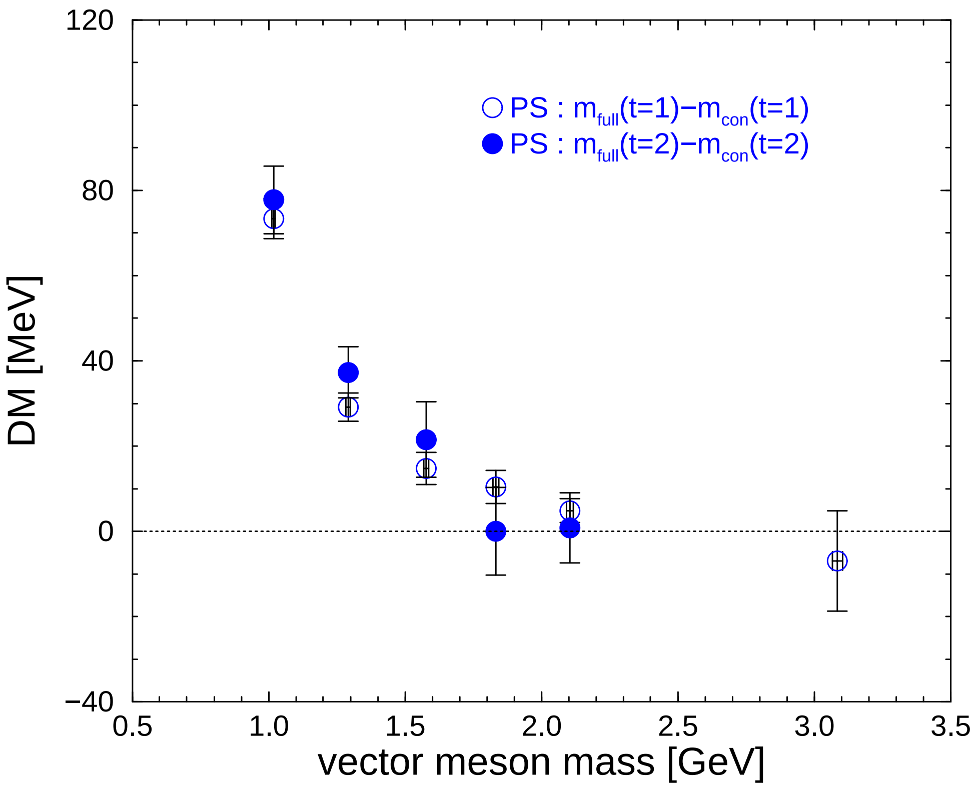
as $a \longrightarrow 0$?

Remember:

$\Delta M(n_f = 0) \approx 90$ MeV

"$I = 0$" vs. "$I = 1$" ???

Disconnected quark line diagrams ? (Quenched: UKQCD, QCD-TARO)

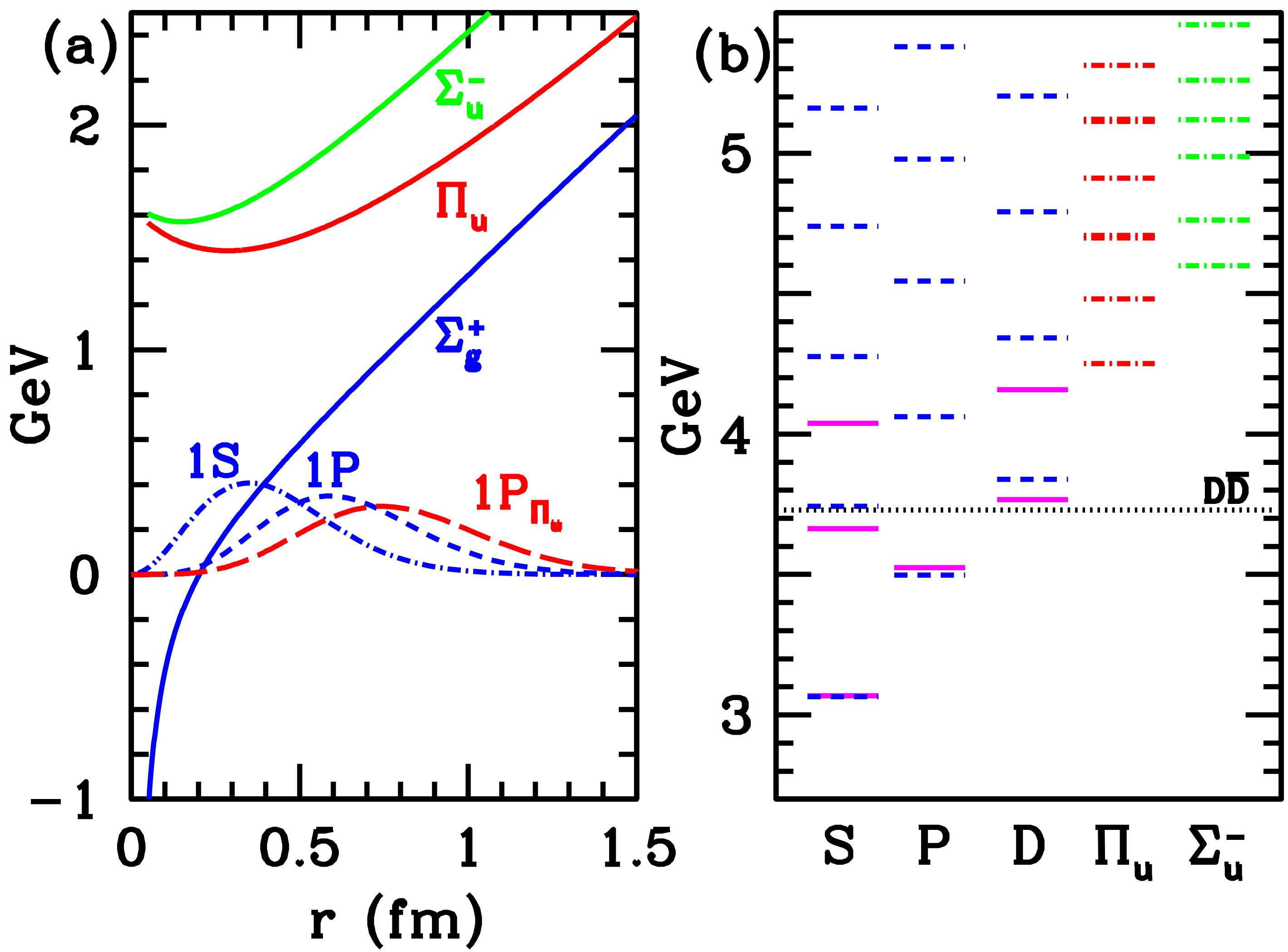


$$DM = m_{\text{“}\eta\text{”}} - m_{\text{“}\pi\text{”}}$$

Disconnected diagrams $\curvearrowright$ $m_\eta > m_\pi$ $\curvearrowright$ $m_\omega - m_\eta < m_\rho - m_\pi$

UKQCD: sign change for heavy quarks ??

Potentials (Juge, Kuti, Morningstar 03):



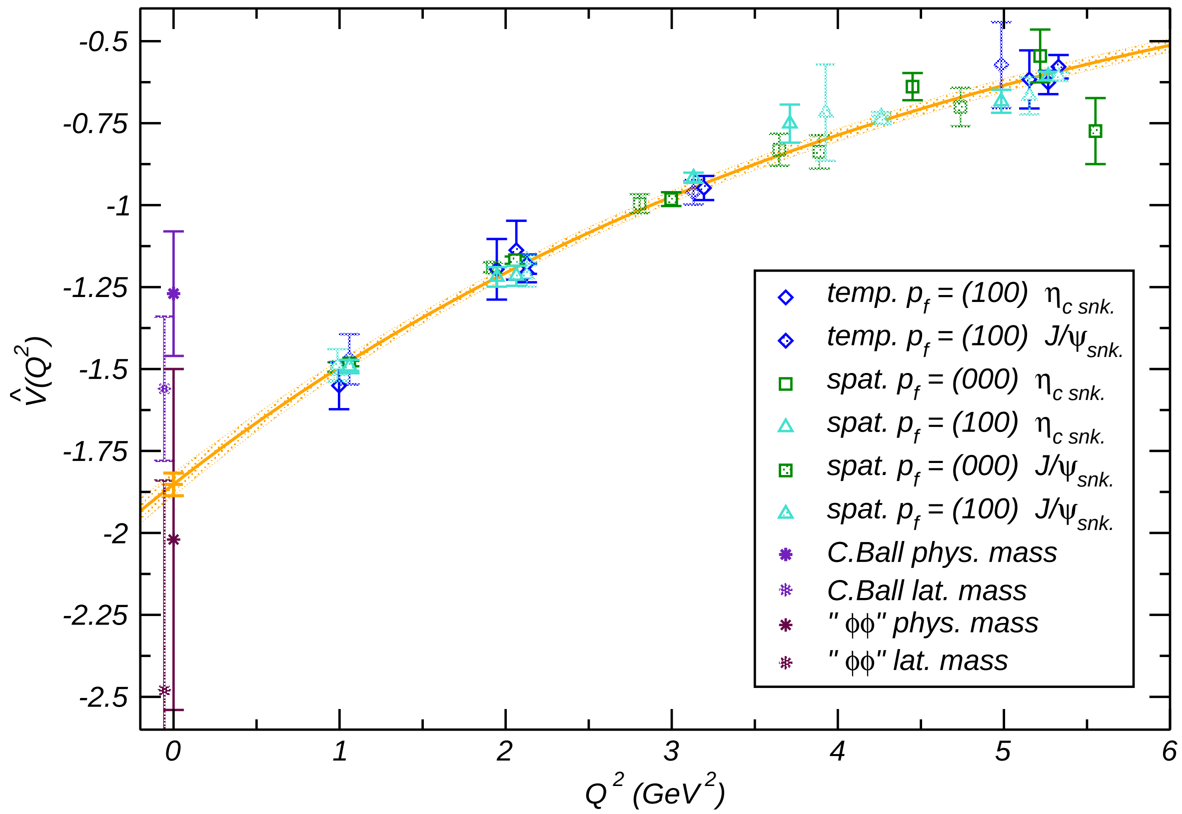
Missing states ? Obviously we have to move beyond spectroscopy !

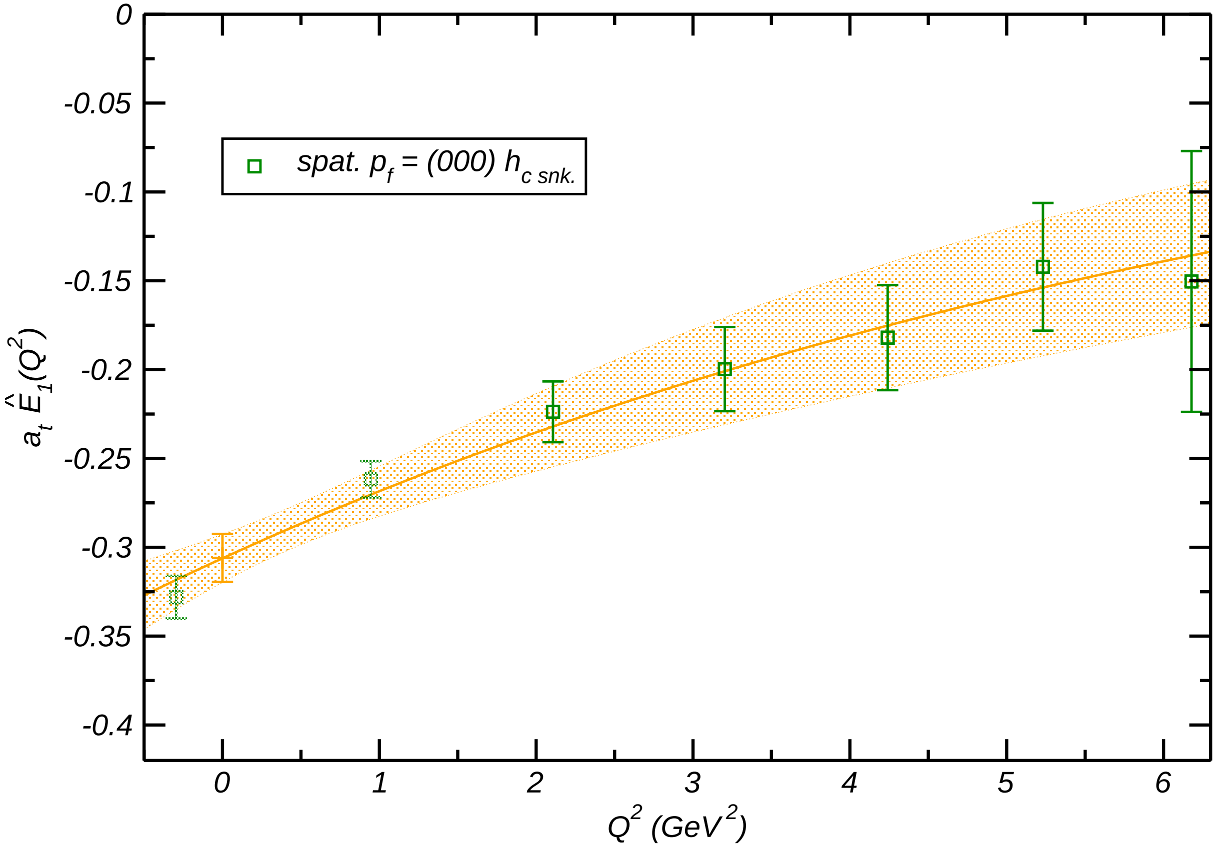
EM decay constants and transition rates

Dudek et al 06

$$\Gamma(J/\psi \to \eta_c \gamma_{M1}) = \frac{\mathbf{q}^3}{(m_{\eta_c} + m_{J/\psi})^2} \frac{64}{27} \alpha_{\mathsf{fs}} |\hat{V}(0)|^2$$

where $\hat{V}(Q^2)$ times kinematic factor $\propto \langle \eta_c(\mathbf{p}') | j^\mu(0) | J/\psi(\mathbf{p}) \rangle$.

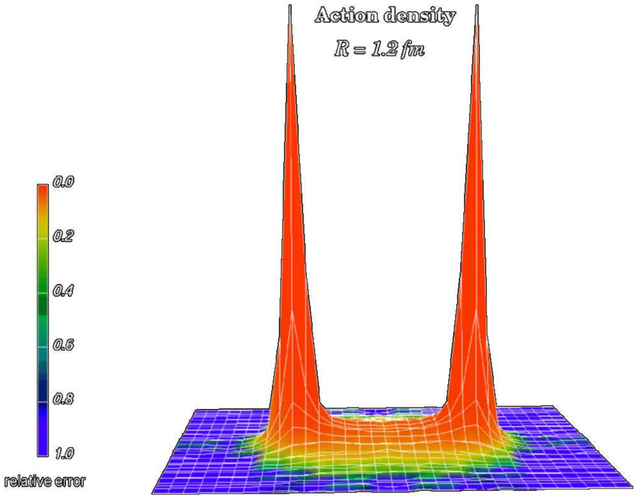


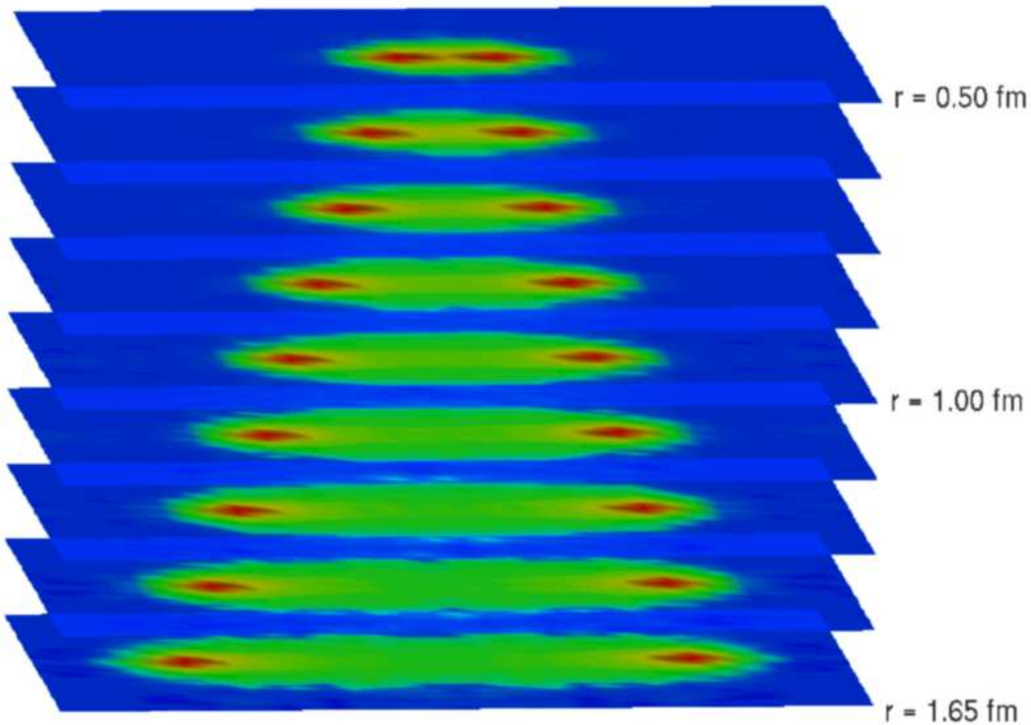


$\curvearrowright$ $\Gamma(h_c \to \eta_c \gamma_{E1,C1}) = \begin{array}{c} 663(132) \\ 601(63) \end{array}$ keV. CLEO: $\psi' \to \pi^0 h_c$, $h_c \to \eta_c \gamma$.

Also have obtained $\chi_{c0} \to J/\psi\gamma_{E1,C1}$ and $\chi_{c1} \to J/\psi\gamma_{E1,M2,C1}$, $f_{J/\psi}, f_{\eta_c}, f_{\psi'}, f_{\eta_c'}$.

The QCD “string” 1995 GB, K. Schilling, C. Schlichter





No sea quarks !!!

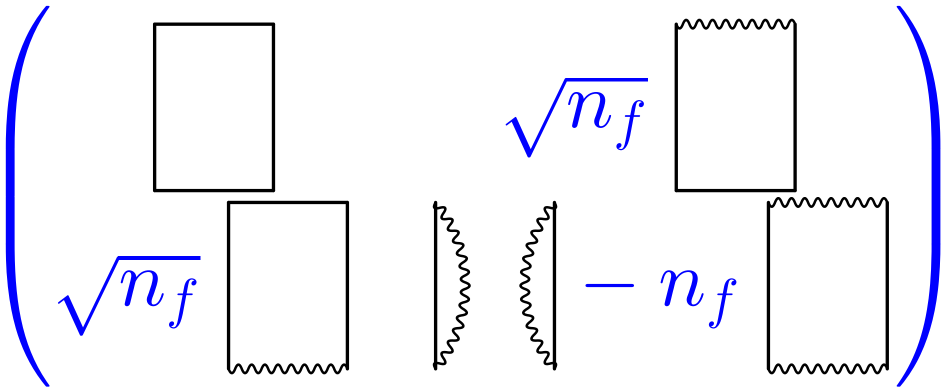
String breaking: GB, H. Neff, T. Düssel, T. Lippert, Z. Prkacin, K. Schilling 04-06

Eigenstates:

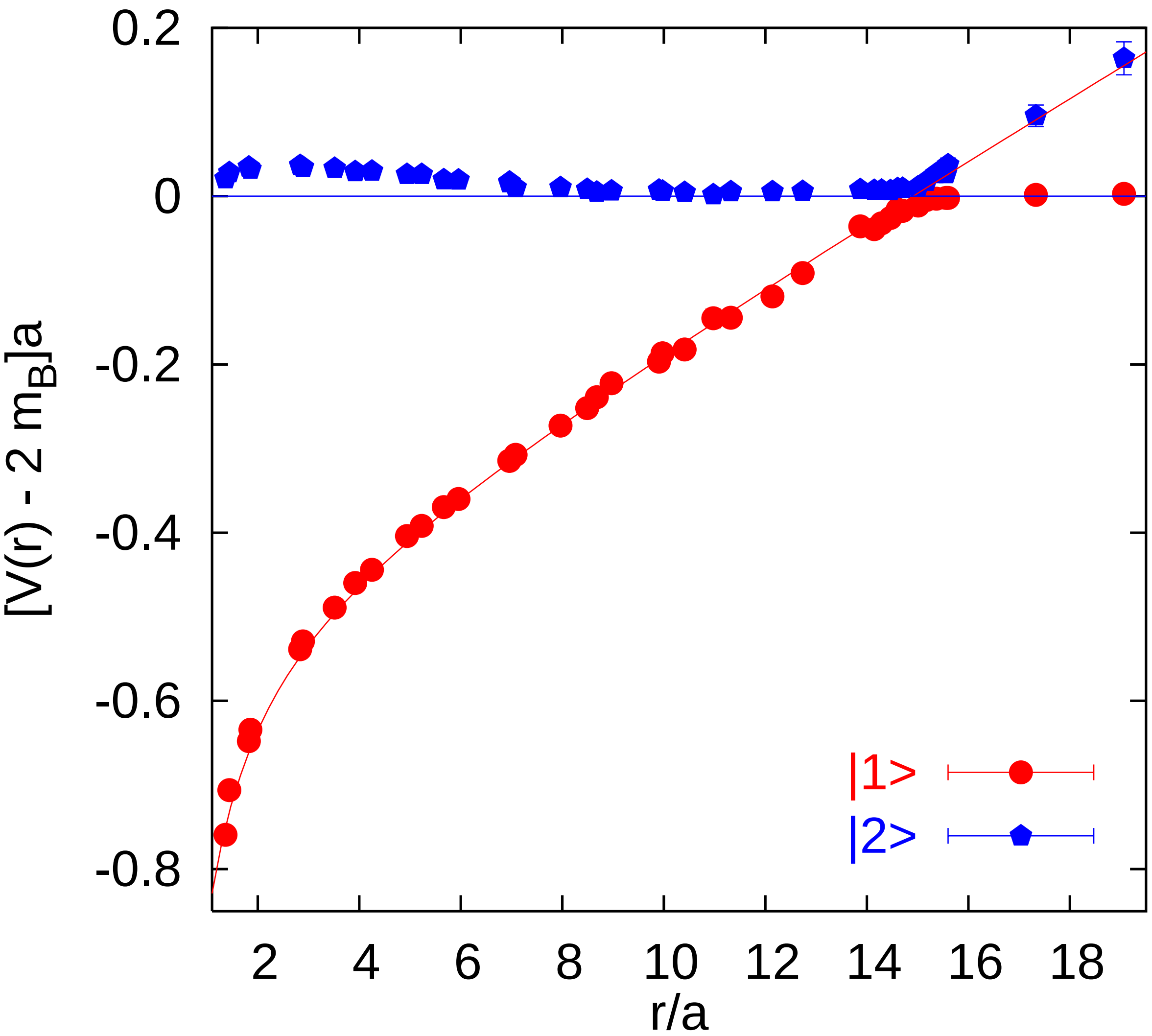
$$
\begin{aligned}
|1\rangle &= \cos\theta\,|\overline{Q}Q\rangle + \sin\theta\,|B\overline{B}\rangle \\
|2\rangle &= -\sin\theta\,|\overline{Q}Q\rangle + \cos\theta\,|B\overline{B}\rangle
\end{aligned}
$$

with $B = \overline{Q}q$.

Correlation matrix:



Static potentials



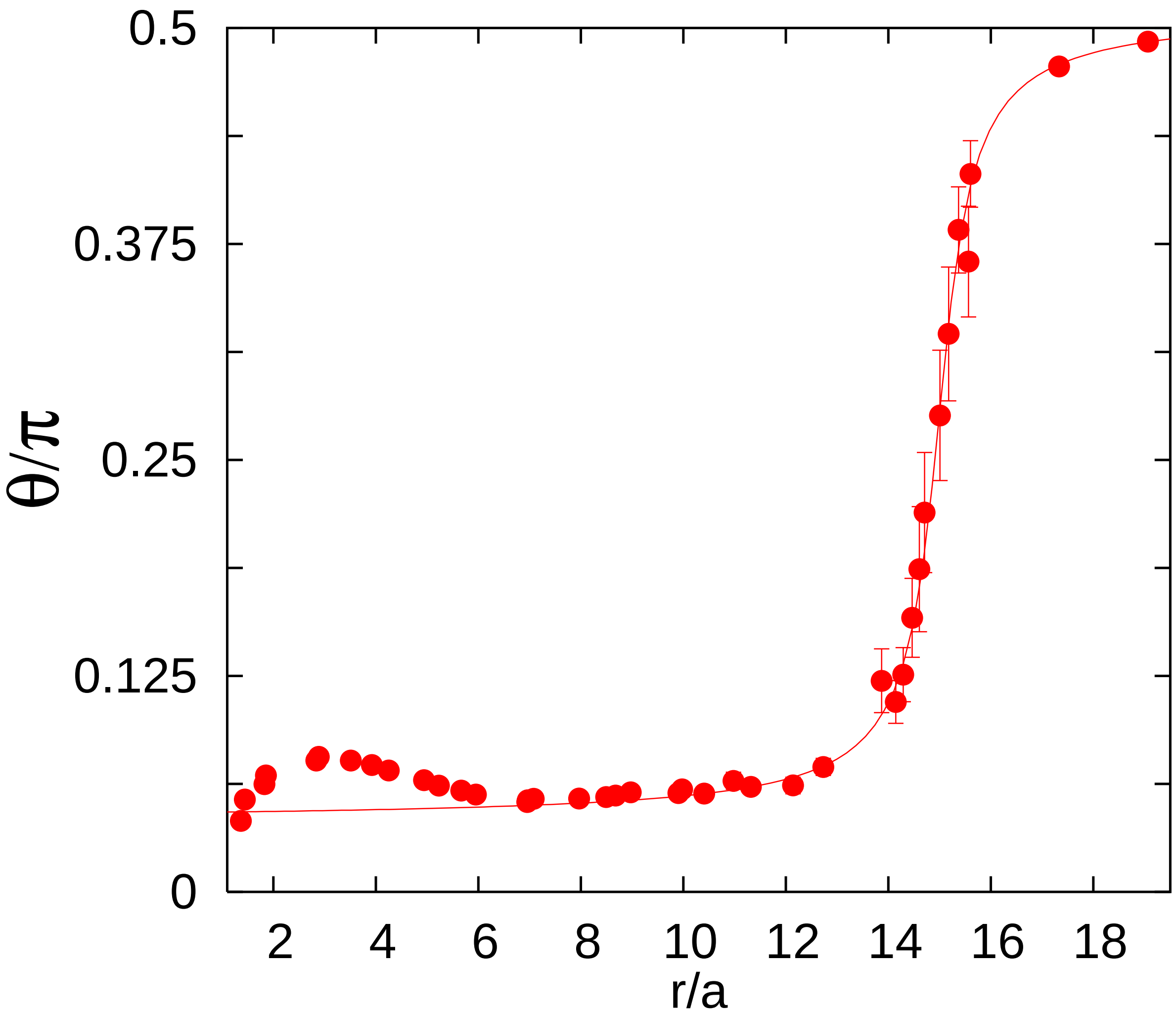
$n_f = 2$ Wilson

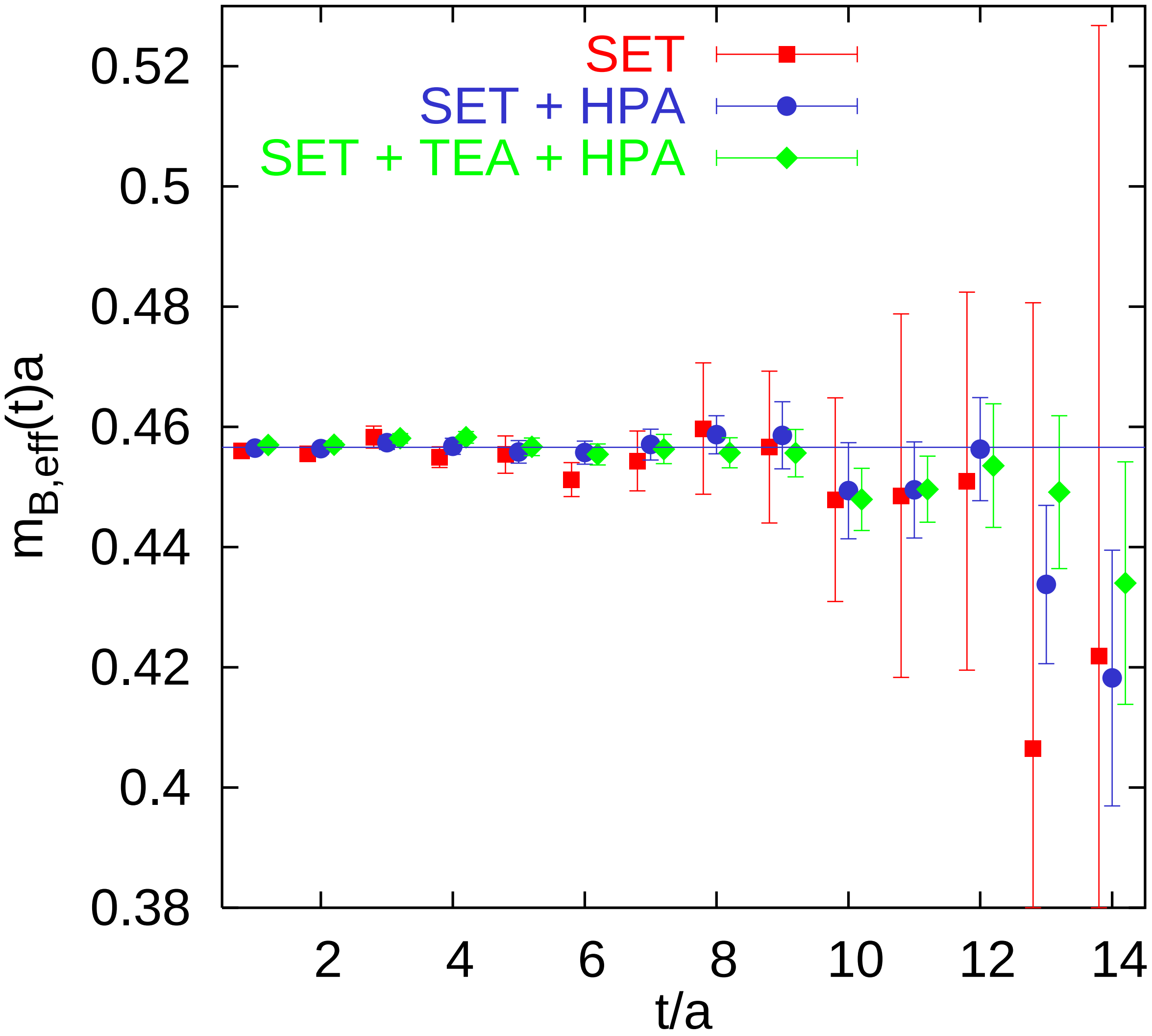
$m_q \approx m_s$

$a \approx 0.083$ fm

$a^{-1} \approx 2.37$ GeV

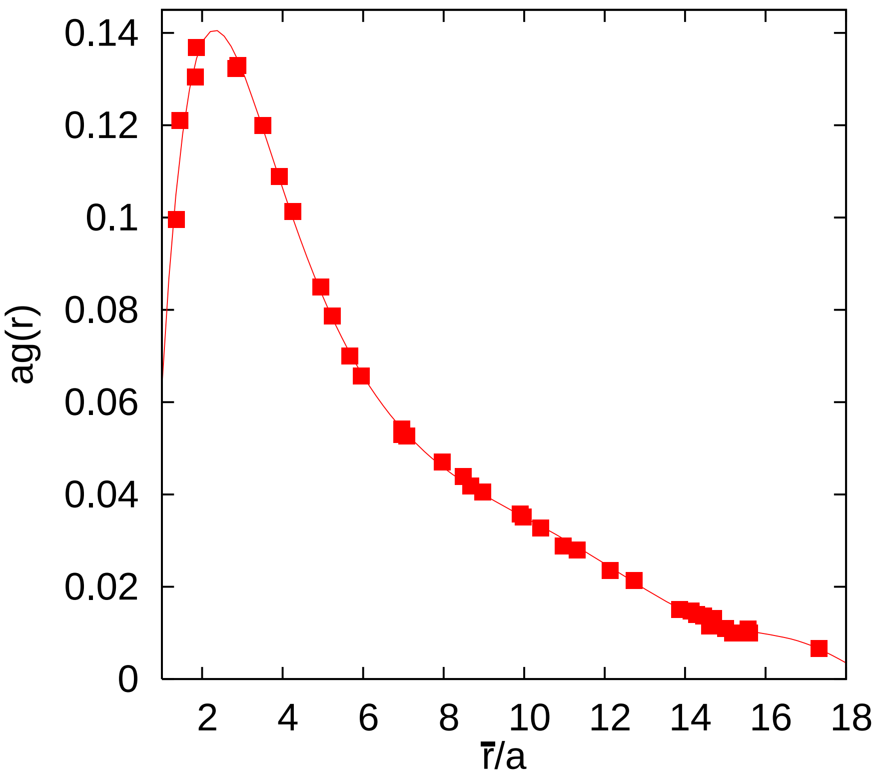
Mixing angle: $B\overline{B}$ content of ground state

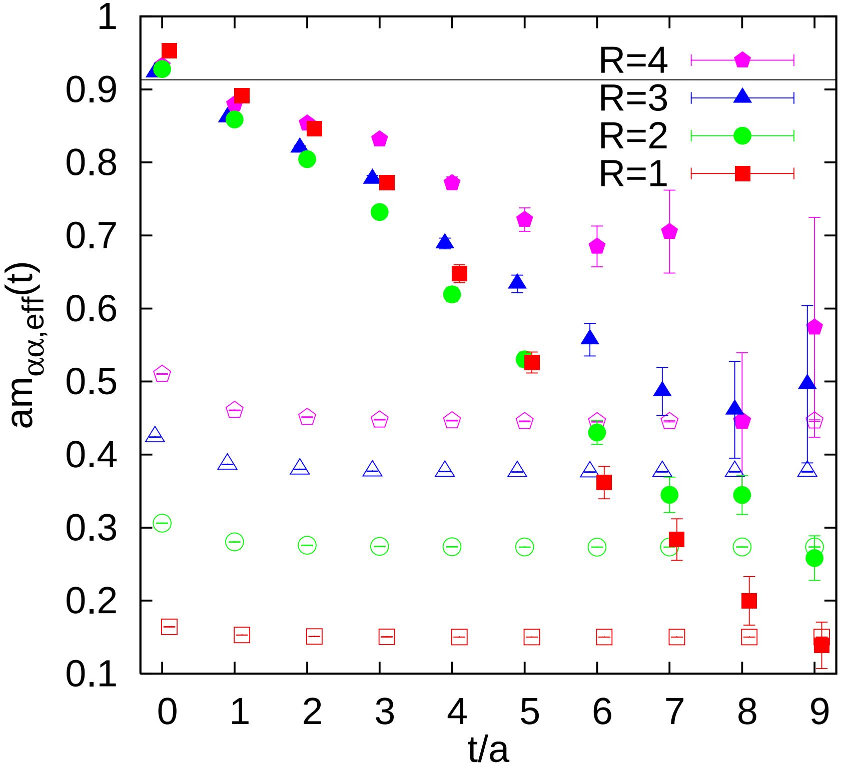




$B = \overline{Q}q$ effective mass: $m_{\text{eff}}(t) = a^{-1} \ln[C(t)/C(t+a)]$

“Decay”: $B\overline{B} \to \overline{Q}Q$ at $r < r_c$, $\overline{Q}Q \to B\overline{B}$ at $r > r_c$

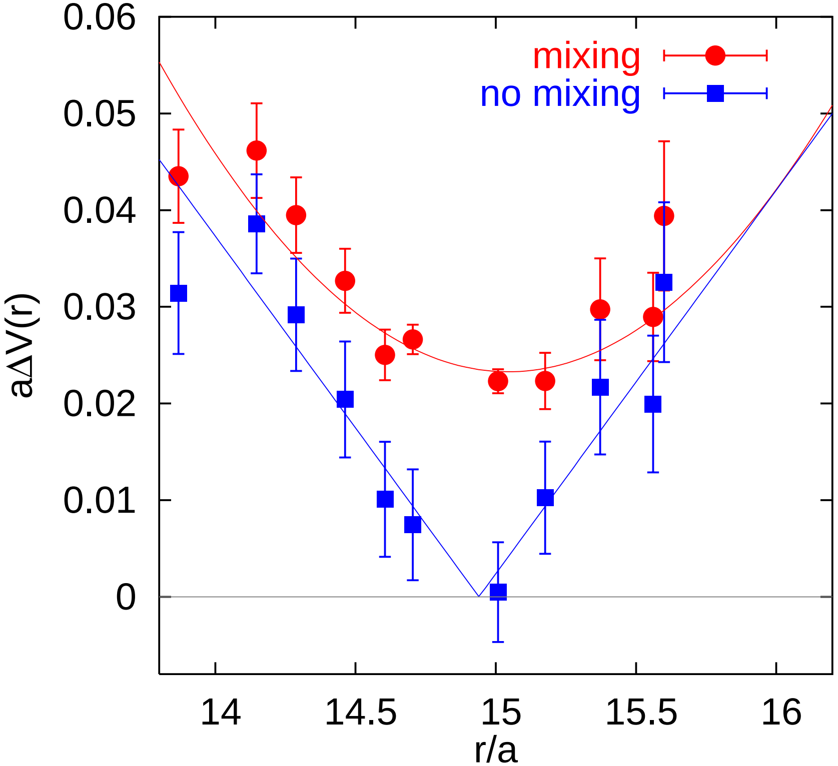




$$g(r) = \Delta E(r)\frac{\sin 2\theta(r)}{2} \propto \frac{dC_{QB}}{dt}$$

$$m_{\text{eff}}(t) = a^{-1}\ln\left[\frac{C(t)}{C(t+a)}\right]$$

Energy gap in the string breaking region

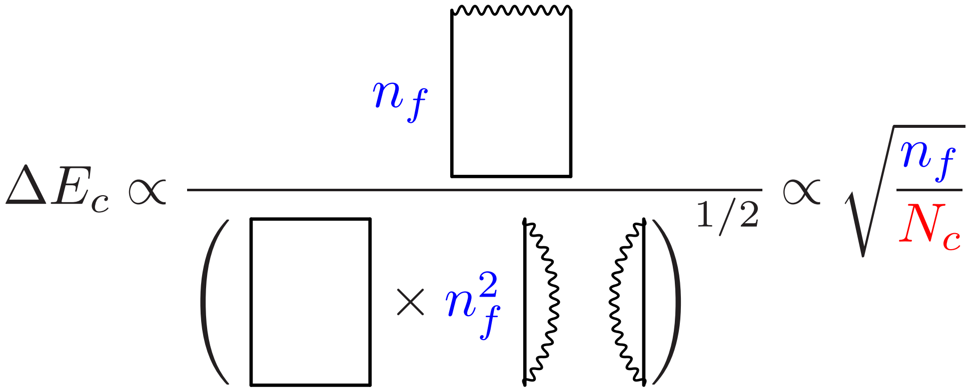


$\Delta E(r_c) \approx 2g(r_c) \approx 51(3)$ MeV

Large N_c:

$SU(N_c)$ QCD $Q\overline{Q} \leftrightarrow (Q\bar{q})(q\overline{Q})$:

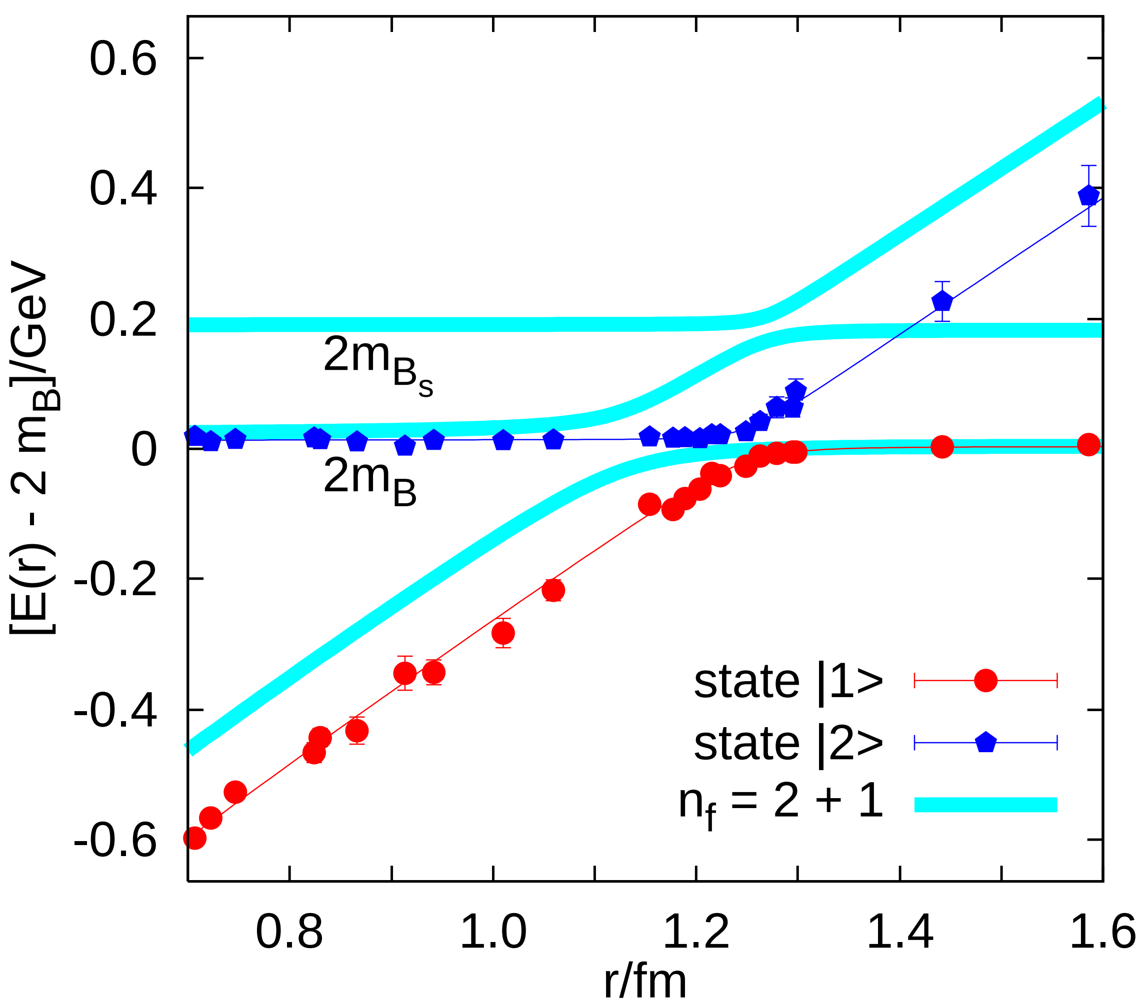
(also $SU(N_c)$ plus n_f fundamental Higgs)

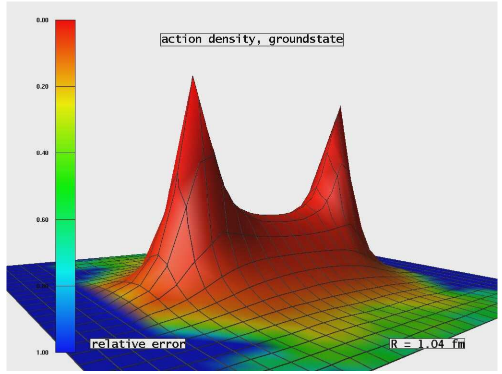


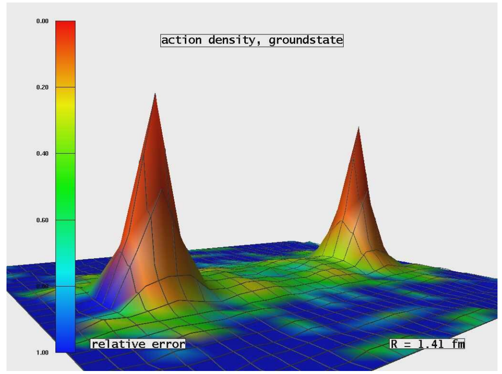
Adjoint potential $\leftrightarrow$ 2 Gluelumps:

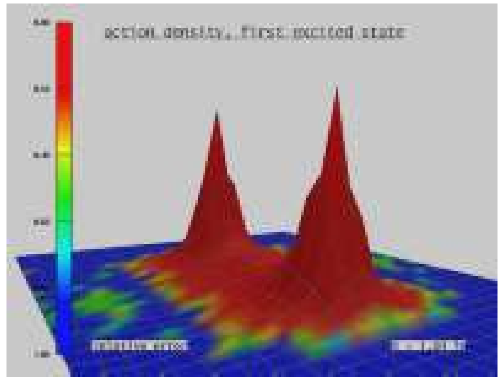
$$\Delta E_c \propto \frac{1}{N_c}$$

String breaking in detail









Quarkonium decay

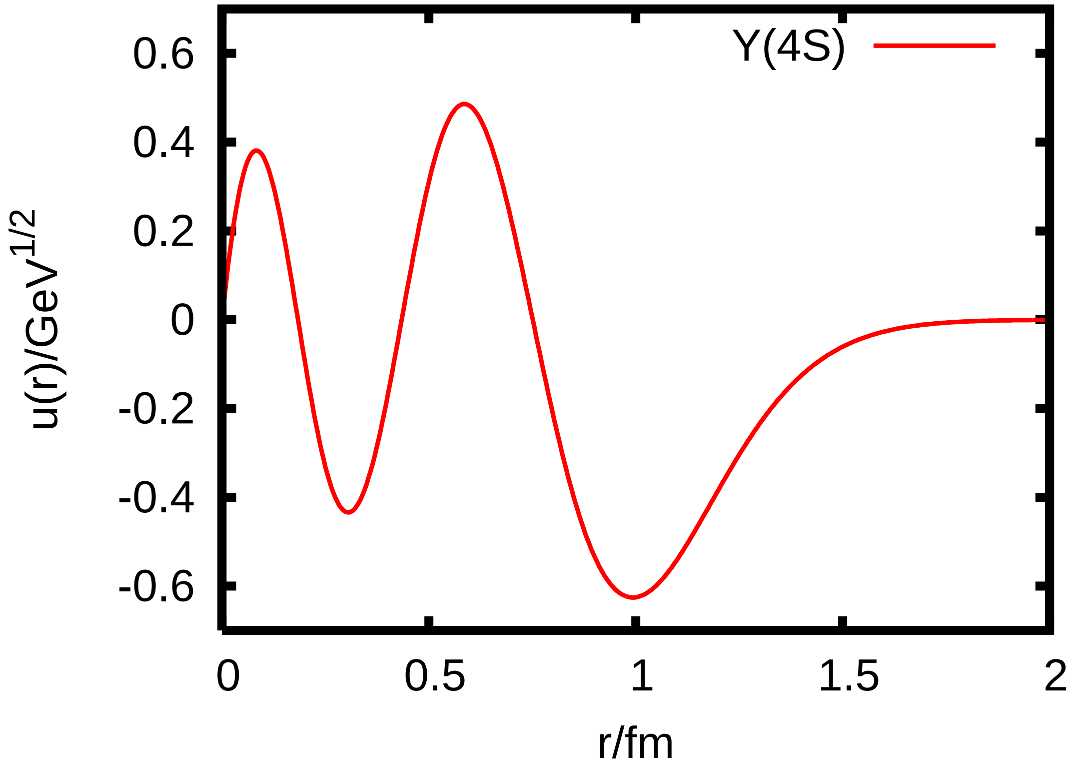
Coupled channel system: $H\psi(\mathbf{r}) = E\psi(\mathbf{r})$ with

$$H = \begin{pmatrix} \frac{1}{m_Q} & 0 \\ 0 & \frac{1}{m_B} \end{pmatrix} \mathbf{p}^2 + V(r) \quad , \quad \psi(\mathbf{r}) = \begin{pmatrix} \psi_{\overline{Q}Q}(\mathbf{r}) \\ \psi_{B\overline{B}}(\mathbf{r}) \end{pmatrix}$$

$$\begin{aligned} V(r) &= \begin{pmatrix} V_{\overline{Q}Q}(r) & g(r) \\ g(r) & V_{B\overline{B}}(r) \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} V_1(r) & 0 \\ 0 & V_2(r) \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \end{aligned}$$

$V_{B\overline{B}}(r) \approx 2M_B, \quad \theta = \theta(r)$

Decay rate: $\Gamma = 2\pi \sum_{i,k} \rho(k) |\langle \mathbf{k}, i | g | \psi_{\overline{Q}Q} \rangle|^2$



$\Gamma(\Upsilon(4S) \to B\overline{B}) \approx 5\,$MeV $\approx$ experiment$/2$.

Effect of 4-quark sector on bound states:

coupled channel analysis:

$H\psi = E\psi$ with $H = (H_0 \oplus H_2) + H_I$, $\psi = \psi_0 \oplus \psi_2$

$$\left[H_0 + H_I^\dagger \frac{1}{E - H_2 + i\epsilon} H_I \right] \psi_0 \approx E\psi_0$$

Molecules:

H_2 includes $Q\bar{q}\,q\overline{Q} \leftrightarrow Q\bar{q}\,q\overline{Q}$ (large r) and $Q\bar{q}\,q\overline{Q} \leftrightarrow Q\overline{Q}\,q\bar{q}$ (small r).

($X \leftrightarrow J/\psi + \omega$ vs. $X \leftrightarrow D\overline{D}^*$).

For charmonium we should go beyond the heavy quark approximation.

Next steps:

$$c\bar{q}q\bar{c} \longleftrightarrow c\bar{c} \longleftrightarrow cg\bar{c}.$$

Sea quark mass dependence.

Summary

$n_F = 0$ charmonium spectrum including hybrids under control.

Is L a (very) good quantum number ?

Is c "light"? $m_c/\overline{\Lambda} \approx 3, v > 0.5$, overlap with glueball spectrum.

Disconnected diagrams have 0–20 MeV effect. First calculations.

First steps towards $n_F = 2 + 1$.

Promising studies of EM decay and transition rates.

To do: study with full $c\bar{c}$, $cg\bar{c}$, $c\bar{q}q\bar{c}$ basis.