The Sivers effect in $p\bar{p}$-interactions

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- Introduction
- Parton densities (forward, p_T-dependent)
- Semi-inclusive processes
- Sivers effect and single-spin asymmetries
- Phenomenology of the Sivers effect
- Drell-Yan measurements in $p\bar{p}$-interactions
- Summary

General strategy

1. Factorization

$$\sigma \propto (\text{pert. part}) \otimes (\text{non-pert. part}) + \mathcal{O}\left(\frac{1}{Q}\right)$$

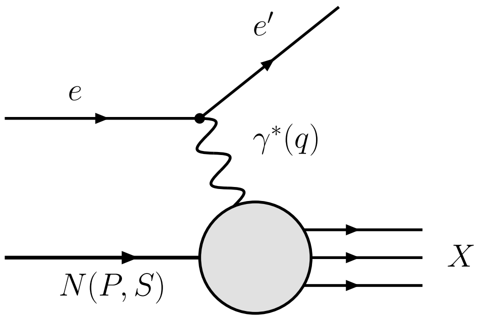
- required for interpretation
- does not hold for each hard process
- required scale: $Q > 1\,\mathrm{GeV}$

2. Universality

- allows one to make predictions
- can be non-trivial

Inclusive deep-inelastic scattering

- Process



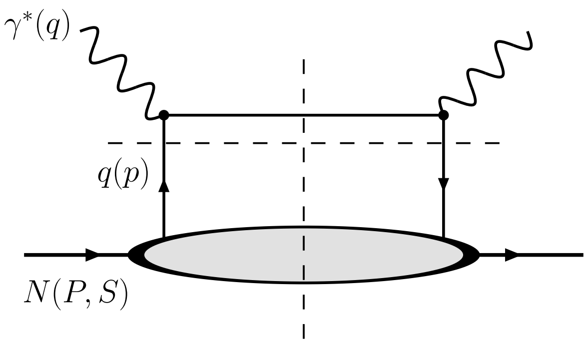
Variables

$$Q^2 = -q^2 > 0 \qquad x = \frac{Q^2}{2P\cdot q} \qquad \Phi_l^S$$

- Optical theorem

$$\sigma_{\gamma^* N \to X} \propto \mathrm{Im}\, A(\gamma^* N \to \gamma^* N, \vartheta = 0)$$

- Parton model



$\longrightarrow$ Factorization

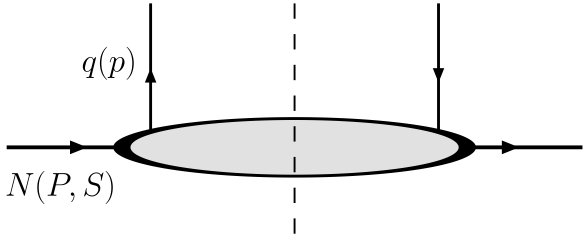
e.g., $f_1^q(x, Q^2)$

$$p^+ = \frac{1}{\sqrt{2}}(p^0 + p^3) = xP^+$$

$$p^- = \frac{1}{\sqrt{2}}(p^0 - p^3)\,, \quad \vec{p}_T = (p^1, p^2)$$

Parton densities

1. Forward parton densities



- Unpolarized pdf

$$f_1(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P,S|\, \bar{\psi}(0)\, \gamma^+\, \psi(\xi^-)\, |P,S\rangle$$

$$\xi^- = \frac{1}{\sqrt{2}}(t-z) \qquad\qquad \xi^+ = \xi_T = 0$$

- Helicity pdf

$$g_1(x) \propto \langle \;|\, \bar{\psi}(0)\, \gamma^+\gamma_5\, \psi(\xi^-)\, |\; \rangle$$

- Transversity pdf

$$h_1(x) \propto \langle \;|\, \bar{\psi}(0)\, i\sigma^{i+}\gamma_5\, \psi(\xi^-)\, |\; \rangle \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

2. p_T-dependent parton densities
(Boer, Mulders, 1998)

$$\int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3}\, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\, \langle P,S|\,\bar{\psi}(0)\,\gamma^+\,\psi(\xi^-,\vec{\xi}_T)\,|P,S\rangle$$

$$= \quad f_1(x,\vec{p}_T^{\,2}) - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x,\vec{p}_T^{\,2})$$

Parameterization of

$$\Phi^{[\Gamma]}(x,\vec{p}_T,S) = \int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3}\, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\, \langle P,S\,|\,\bar{\psi}(0)\,\Gamma\,\psi(\xi^-,\vec{\xi}_T)\,|\,P,S\rangle$$

Leading order (8 functions, 2 T-odd)

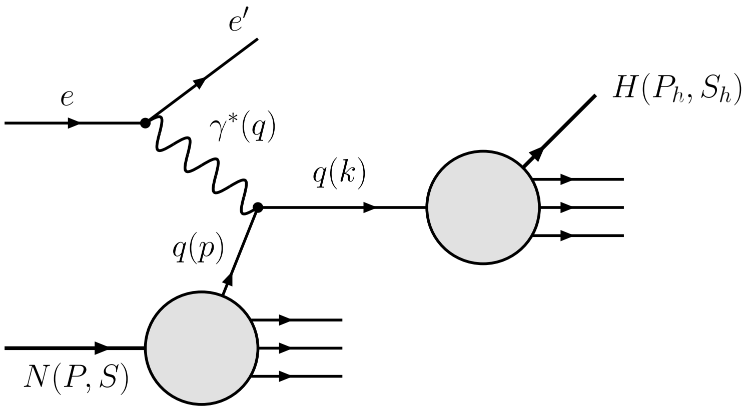
$$\Phi^{[\gamma^+]} \quad = \quad f_1 - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M}\, f_{1T}^{\perp}$$

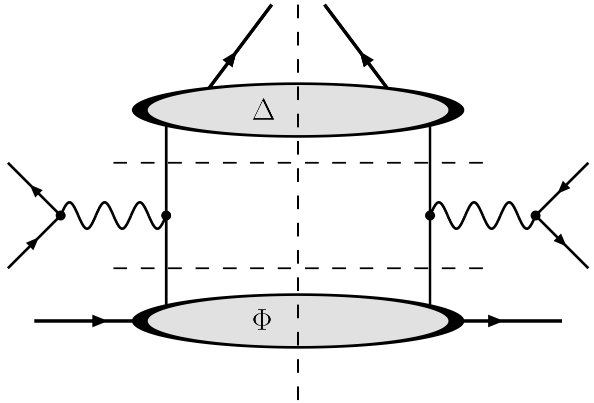
$$\Phi^{[\gamma^+\gamma_5]} \quad = \quad \lambda\, g_{1L} + \frac{\vec{p}_T\cdot\vec{S}_T}{M}\, g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma_5]} \quad = \quad S_T^i\, h_{1T} + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^{\perp} + \frac{\vec{p}_T\cdot\vec{S}_T}{M}\, h_{1T}^{\perp}\right) - \frac{\varepsilon_T^{ij} p_{Tj}}{M}\, h_1^{\perp}$$

Semi-inclusive processes

1. Semi-inclusive deep-inelastic scattering $e\,N \to e'\,H\,X$





$\sigma \propto \hat{\sigma}_{pert} \otimes \Phi \otimes \Delta$

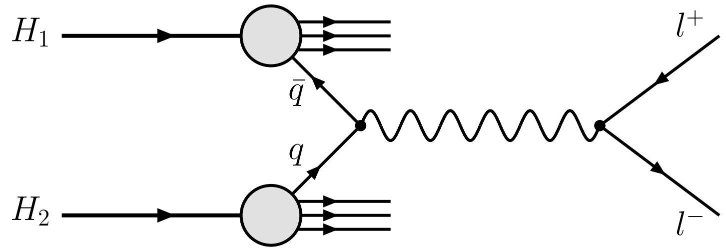
Factorization formula depends on kinematics

- σ integrated upon P_{hT}
- σ differential in P_{hT} and $P_{hT} \approx Q$
- σ differential in P_{hT} and $P_{hT} \ll Q$

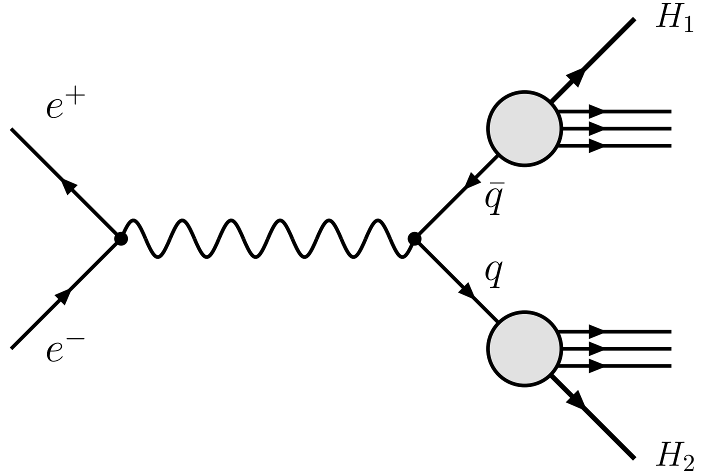
$$\sigma \propto \hat{\sigma}_{pert} \otimes \mathrm{pdf} \otimes \mathrm{ff} \otimes \mathrm{soft}$$

(Collins, Soper, 1981; Collins, Soper, Sterman, 1985;
Ji, Ma, Yuan, 2004; Collins, Metz, 2004)

2. Drell-Yan process $H_1\, H_2 \to l^+\, l^-\, X$



3. Electron-positron annihilation $e^+\, e^- \to H_1\, H_2\, X$



T-odd parton densities

- (Sivers, 1990)

$$\int \frac{d\xi^- d^2\vec{\xi}_T}{2(2\pi)^3}\, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)}\, \langle P, \uparrow |\, \bar{\psi}(0)\, \gamma^+\, \psi(\xi^-, \vec{\xi}_T)\, | P, \uparrow \rangle$$

$$= \quad f_1(x, \vec{p}_T^{\,2}) - \frac{(\vec{p}_T \times \vec{S}_T)\cdot \hat{P}}{M} f_{1T}^{\perp}(x, \vec{p}_T^{\,2})$$

$\longrightarrow$ single-spin/azimuthal asymmetry

- (Boer, Mulders, 1998)

$$h_1^{\perp}(x, \vec{p}_T^{\,2}) \propto \langle P, \mathrm{unp}\, |\, \bar{\psi}(0)\, i\sigma^{i+}\gamma_5\, \psi(\xi^-, \vec{\xi}_T)\, |\, P, \mathrm{unp} \rangle$$

Time-reversal:

$f_{1T}^{\perp} = -f_{1T}^{\perp} = 0 \qquad\qquad h_1^{\perp} = 0$

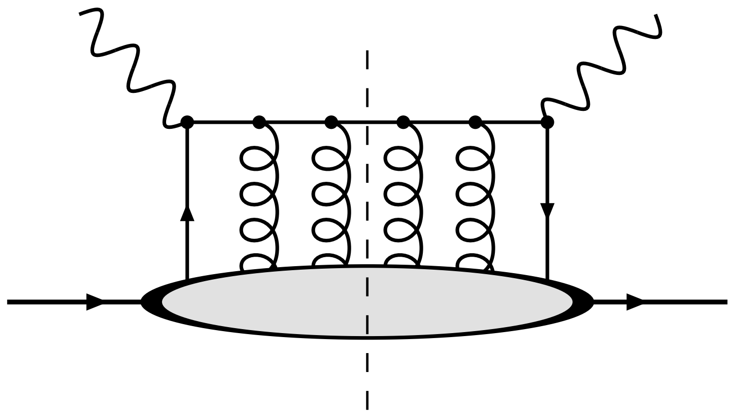
Color gauge invariance

1. Forward partonic functions

$$\int d\xi^- \, e^{ixP^+\xi^-} \langle \, | \, \bar{\psi}(0) \, \Gamma \, \mathcal{L}(0;\xi^-) \, \psi(\xi^-) \, | \, \rangle$$

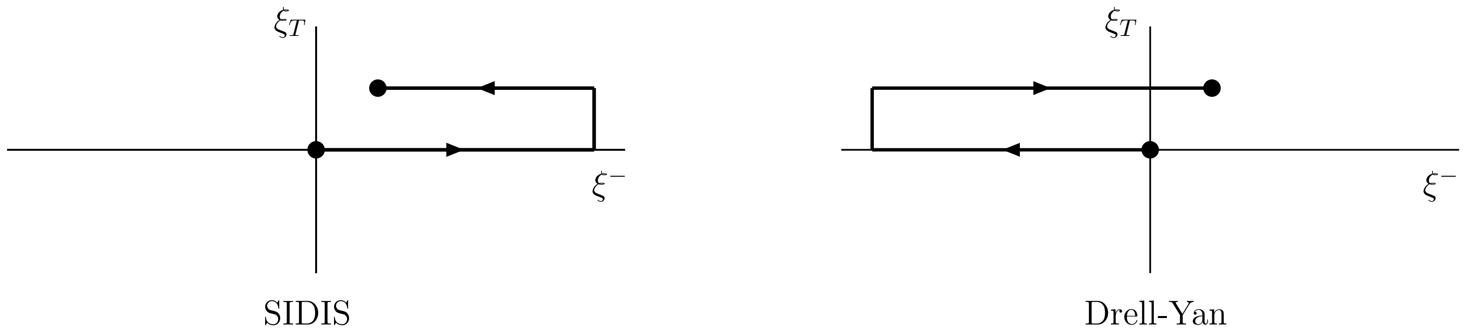
$$\mathcal{L}(0;\xi^-) = \mathcal{P} \exp\left(-ig \int_0^{\xi^-} d\eta^- \, A^+(0,\eta^-,\vec{0}_T) \right)$$

Gauge-link generated by rescattering



2. p_T-dependent partonic functions

$$\int d\xi^- d^2\vec{\xi}_T \, e^{i(xP^+\xi^- - \vec{p}_T\cdot\vec{\xi}_T)} \, \langle \, | \, \bar{\psi}(0) \, \Gamma \, \mathcal{L}(0, \vec{0}_T; \xi^-, \vec{\xi}_T) \, \psi(\xi^-, \vec{\xi}_T) \, | \, \rangle$$



- $\mathcal{L}(0, \vec{0}_T; \xi^-, \vec{\xi}_T) = \mathcal{L}(0, \vec{0}_T; \infty, \vec{0}_T) \times \mathcal{L}(\infty, \vec{0}_T; \infty, \vec{\xi}_T) \times \mathcal{L}(\infty, \vec{\xi}_T; \xi^-, \vec{\xi}_T)$
  (Belitsky, Ji, Yuan, 2002)

- Gauge-link $\longrightarrow$ T-odd pdfs can be non-zero
  (Brodsky, Hwang, Schmidt, 2002; Collins, 2002)

- Different links for semi-inclusive DIS and Drell-Yan $\longrightarrow$ Universality?
  Time-reversal: $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ $\qquad$ $h_1^{\perp}\big|_{DY} = -h_1^{\perp}\big|_{DIS}$
  (Collins, 2002)

Universality of non-perturbative factors

1. Problem
   - Semi-inclusive DIS

$$\sigma\big|_{DIS} \propto \hat{\sigma}_{pert} \otimes \mathrm{pdf} \otimes \mathrm{ff} \otimes \mathrm{soft}$$

   - Drell-Yan

$$\sigma\big|_{DY} \propto \hat{\sigma}_{pert} \otimes \mathrm{pdf} \otimes \mathrm{pdf} \otimes \mathrm{soft}$$

$$\mathrm{pdf}\big|_{DIS} \overset{?}{=} \mathrm{pdf}\big|_{DY}$$

$$\mathrm{soft}\big|_{DIS} \overset{?}{=} \mathrm{soft}\big|_{DY}$$

2. Results
   - Parton densities:

     Time-reversal: $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ $\qquad h_{1}^{\perp}\big|_{DY} = -h_{1}^{\perp}\big|_{DIS}$

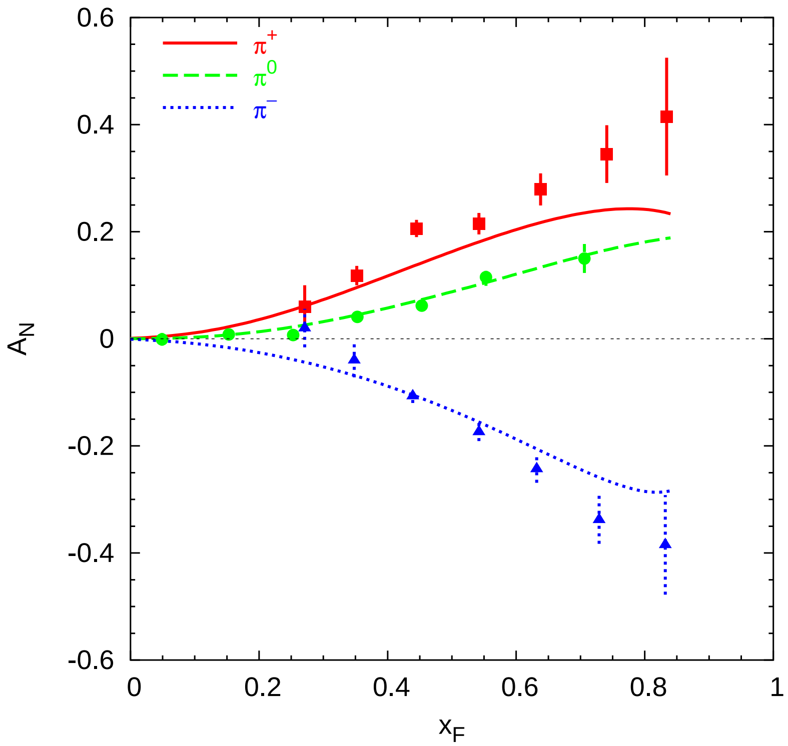
     6 T-even pdfs are universal

   - Soft factor:

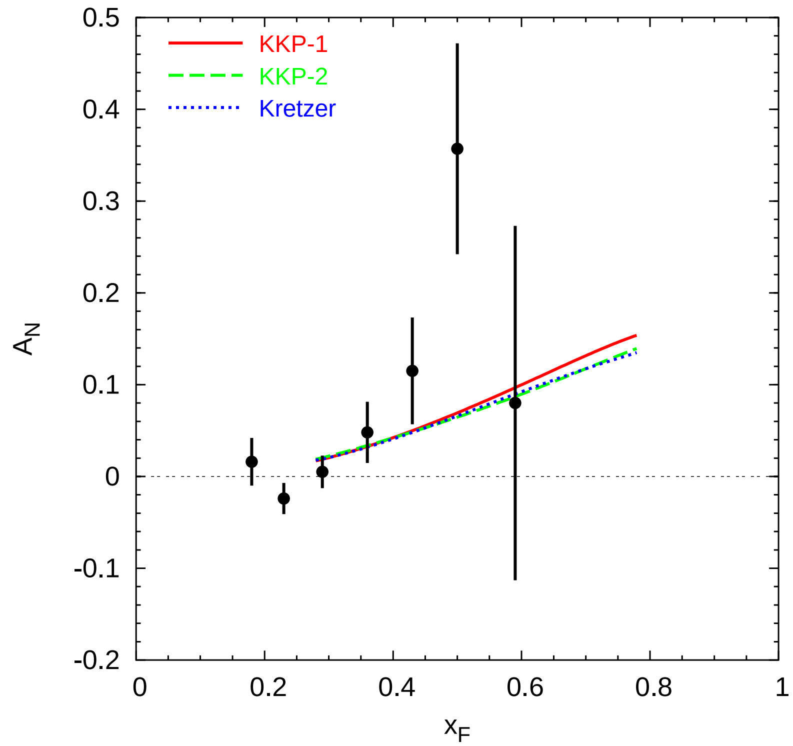
     Time-reversal and analytical structure: $\mathrm{soft}\big|_{DIS} = \mathrm{soft}\big|_{DY}$
     (Metz 2002; Collins, Metz, 2004)

Transverse SSA in $p\,p^{\uparrow} \;\to \pi\, X$

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \qquad\qquad x_F = \frac{2P_{hL}}{\sqrt{s}}$$



FermiLab, E704, 1990 $\quad \sqrt{s} = 20\,\mathrm{GeV}$



RHIC, STAR, 2004 $\quad \sqrt{s} = 200\,\mathrm{GeV}$

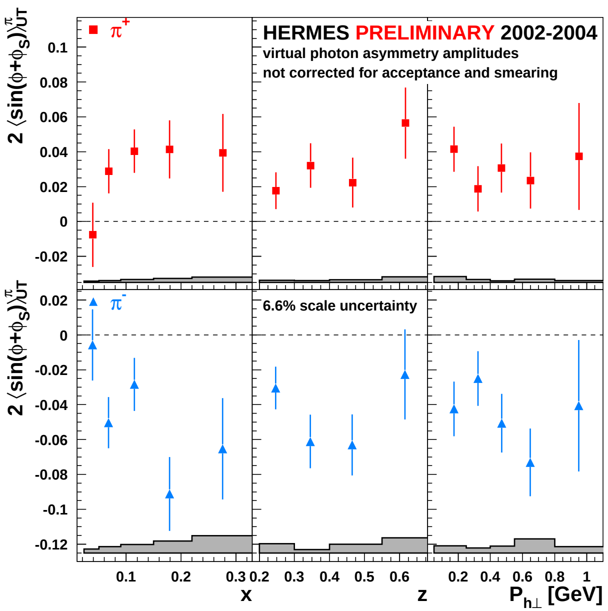
SSA vanishes in leading twist collinear pQCD-description
$\longrightarrow$ (effective) p_T-dependent formalism, etc.

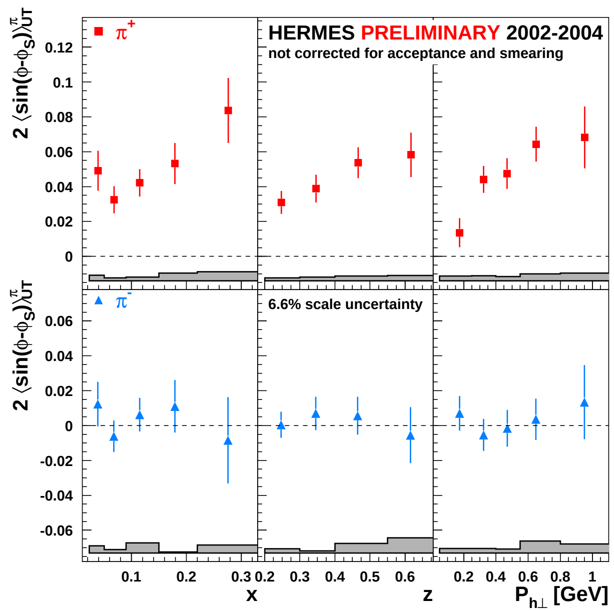
SSAs in semi-inclusive DIS

1. Longitudinal SSAs (twist-3, $\propto 1/Q$)
   - Target spin asymmetry A_{UL}: data from HERMES
   - Beam spin asymmetry A_{LU}: data from CLAS

2. Transverse SSA (leading twist)

$$A_{UT} \propto \sin(\Phi_h + \Phi_S)\, h_1(x)\, H_1^{\perp}(z) + \sin(\Phi_h - \Phi_S)\, f_{1T}^{\perp}(x)\, D_1(z)$$
$$+ \sin(3\Phi_h + \Phi_S) \ldots + \mathcal{O}(1/Q)$$





Also COMPASS data on $l\, D^{\uparrow} \to l'\, H\, X$

Phenomenology of the Sivers effect

(Collins, Efremov, Goeke, Grosse-Perdekamp, Menzel, Meredith, Metz, Schweitzer, 2004, 2005)

Focus on Sivers effect in semi-inclusive DIS and Drell-Yan
Effect also in $p^{\uparrow}\,\bar{p} \to H\,X$, $p^{\uparrow}\,\bar{p} \to H_1\,H_2\,X$ etc. (theoretically more involved)

1. Extraction of Sivers function $f_{1T}^{\perp}$ from data on A_{UT} $(e\,p^{\uparrow} \to e\,\pi\,X)$

   Input/approximations:

   - HERMES data

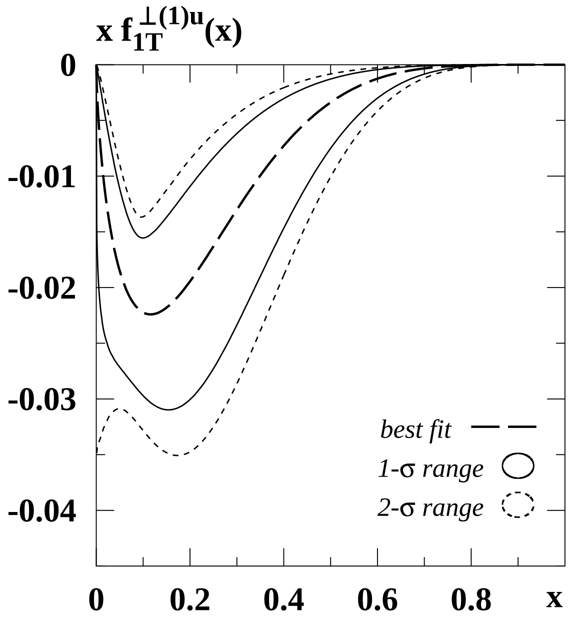
   - Neglect of soft gluon emission

   - Gaussian p_T-behaviour, e.g.,

$$f_1(x, \vec{p}_T^{\,2}) = \frac{1}{\pi\,\langle p_T^2\rangle}\, f_1(x)\, e^{-\vec{p}_T^{\,2}/\langle p_T^2\rangle}$$

   - Exact result in large N_c-limit (Pobylitsa, 2003):

$$f_{1T}^{\perp u}(x, \vec{p}_T^{\,2}) = -f_{1T}^{\perp d}(x, \vec{p}_T^{\,2})$$



$$f_{1T}^{\perp(1)}(x) \equiv \int d^2\vec{p}_T\, \frac{\vec{p}_T^{\,2}}{2M^2}\, f_{1T}^{\perp}(x, \vec{p}_T^{\,2})$$

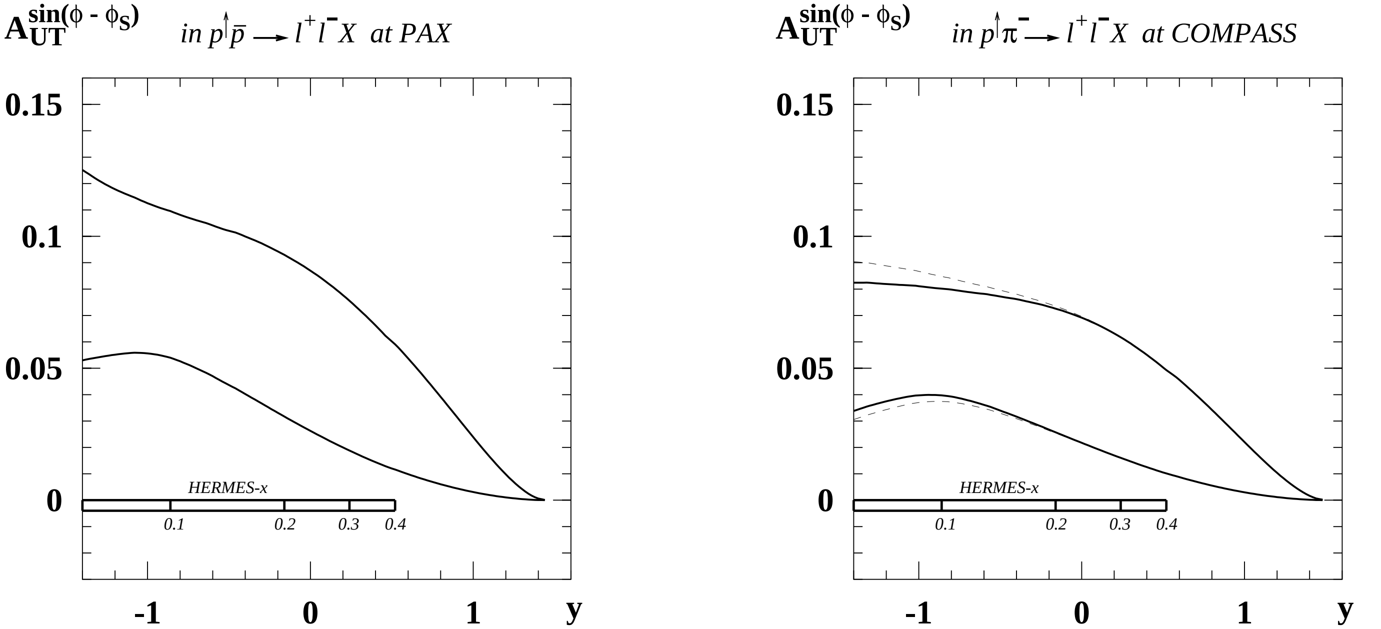
Extractions also by

- Anselmino, Boglione, D' Alesio, Kotzinian, Murgia, Prokudin, 2005
- Vogelsang, Yuan, 2005

Comparison in: Anselmino et al., hep-ph/0511017

2. Calculation of the Sivers asymmetry for Drell-Yan

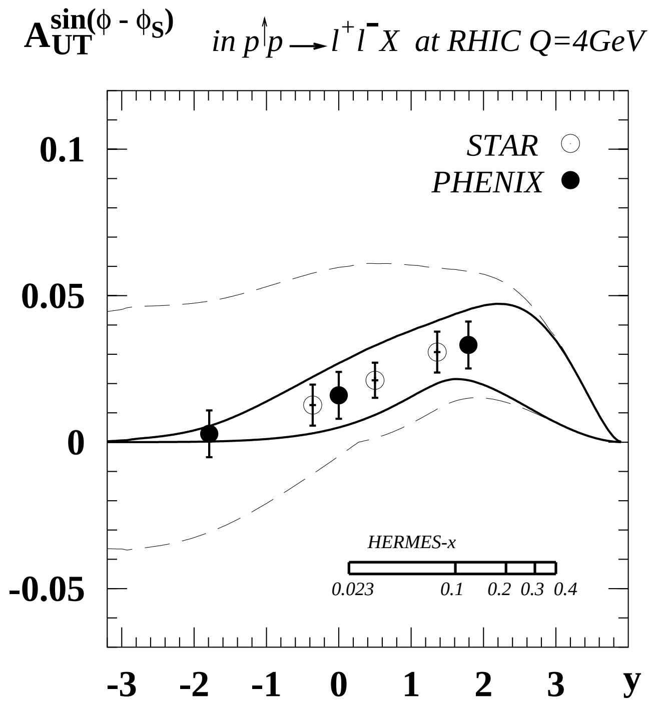
$$y = \frac{1}{2} \ln \frac{x_1}{x_2} \qquad f_{1T}^{\perp(1)\bar{q}}(x) = \pm \frac{f_1^{\bar{u}}(x) + f_1^{\bar{d}}(x)}{f_1^{u}(x) + f_1^{d}(x)} f_{1T}^{\perp(1)q}(x)$$



GSI $p^\uparrow \bar{p} \to l^+ l^- X$ COMPASS $p^\uparrow \pi^- \to l^+ l^- X$

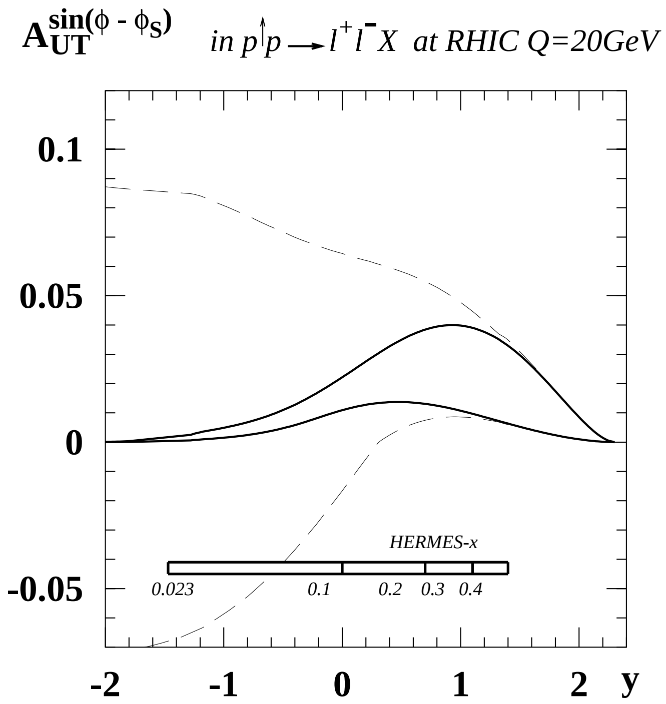
$s = 45\,\mathrm{GeV}^2 \quad Q^2 = 6.25\,\mathrm{GeV}^2$ $s = 400\,\mathrm{GeV}^2 \quad Q^2 = 20\,\mathrm{GeV}^2$

$\longrightarrow$ Prediction $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ can be checked experimentally



RHIC $p^{\uparrow}\, p \to l^+\, l^-\, X$

$\sqrt{s} = 200\,\mathrm{GeV} \quad Q^2 = 16\,\mathrm{GeV}^2$



RHIC $p^{\uparrow}\, p \to l^+\, l^-\, X$

$\sqrt{s} = 200\,\mathrm{GeV} \quad Q^2 = 400\,\mathrm{GeV}^2$

$\longrightarrow$ In pp-collisions strong sensitivity to $f_{1T}^{\perp \bar{q}}$

Drell-Yan measurements in $p\bar{p}$-interactions

1. Double-polarized
   - Transverse polarization:

$$A_{TT}^{p\bar{p}} = a_{TT} \frac{\sum_q e_q^2 \big(h_1^{q/p}(x_1, Q^2) h_1^{\bar{q}/\bar{p}}(x_2, Q^2)\big) + \big(x_1 \leftrightarrow x_2\big)}{\sum_q e_q^2 \big(f_1^{q/p}(x_1, Q^2) f_1^{\bar{q}/\bar{p}}(x_2, Q^2)\big) + \big(x_1 \leftrightarrow x_2\big)}$$

   $\longrightarrow$ Unique tool to measure transversity h_1

   - Longitudinal polarization: $A_{LL}^{p\bar{p}} \propto g_1^{q/p}\, g_1^{\bar{q}/\bar{p}}$

2. Single-polarized
   - Transverse polarization:

$$A_{TU} \propto \sin(\Phi_q - \Phi_S)\, f_{1T}^{\perp}(x_1)\, f_1(x_2) + \sin(\Phi_q + \Phi_S)\, h_1(x_1)\, h_1^{\perp}(x_2) + \ldots$$

   $\longrightarrow$ Access to $f_{1T}^{\perp}$ and $h_1^{\perp}$

   - Longitudinal polarization: $A_{LU} \propto \sin(2\Phi_q)\, h_{1L}^{\perp}(x_1)\, h_1^{\perp}(x_2)$

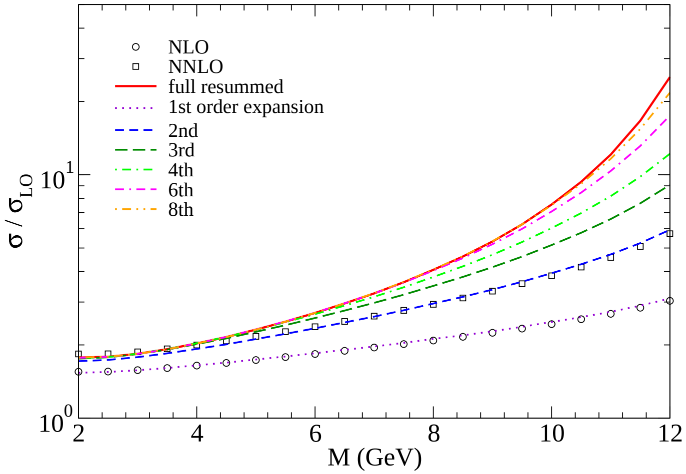
3. Unpolarized

- Unpolarized cross section: fixed order vs. resummation

$$z = \frac{Q^2}{x_1 x_2 s} = \frac{Q^2}{\hat{s}} \quad \begin{cases} = 1 & \text{at tree level} \\ < 1 & \text{at } \mathcal{O}(\alpha_s) \text{ and beyond} \end{cases}$$

k-loop calculation: $\quad \alpha_s^k \left(\frac{\ln^{2k-1}(1-z)}{1-z} \right)_+$

$\sqrt{s} = 14.5\,\text{GeV}$ (Shimizu, Sterman, Vogelsang, Yokaya, 2005)



- Angular distribution

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\Theta + \mu \sin 2\Theta \cos\Phi + \frac{\nu}{2} \sin^2\Theta \cos 2\Phi$$

Collinear picture at $\mathcal{O}(\alpha_s)$: $\lambda + 2\nu = 1$
(Lam, Tung, 1978)

(1) hardly modified at $\mathcal{O}(\alpha_s^2)$
(2) strongly violated by data, ν large

Possible explanation:
(Boer, 1999; Boer, Brodsky, Hwang, 2002; ...)

$$\nu \propto h_1^\perp(x_1)\, h_1^\perp(x_2)$$

Relation of T-odd pdfs to generalized parton distributions:
(Burkardt, 2004, 2005, 2006)

$$f_{1T}^\perp \propto E \qquad\qquad h_1^\perp \propto 2H_T + E_T$$

Summary

- Sivers function can be non-zero

- Universality of non-perturbative factors
  1. $f_{1T}^{\perp}\big|_{DY} = -f_{1T}^{\perp}\big|_{DIS}$ $\qquad h_1^{\perp}\big|_{DY} = -h_1^{\perp}\big|_{DIS}$
  2. $\text{soft}\big|_{DIS} = \text{soft}\big|_{DY}$

- Sign-reversal of Sivers function can be checked in $p^{\uparrow}\,\bar{p} \longrightarrow l^{+}\,l^{-}\,X$ and/or $p\,\bar{p}^{\uparrow} \longrightarrow l^{+}\,l^{-}\,X$

- Drell-Yan measurements (DY is cleanest hard $H\,H$-collision process)
  1. Information on nucleon structure: $h_1\,,\quad g_1\,,\quad f_{1T}^{\perp}\,,\quad h_1^{\perp}\,,\ldots$
  2. Application of pQCD to $p\,\bar{p} \longrightarrow l^{+}\,l^{-}\,X$, $H\,H \longrightarrow \ldots$

Dedicated Drell-Yan program for proton-antiproton interactions would be valuable!