

Towards the hadron spectrum  
using spatially-extended operators  
in lattice QCD

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and their relevance to QCD

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# Lattice Hadron Physics Collaboration

- charge from Nathan Isgur to use Monte Carlo method to extract spectrum of baryon resonances (Hall B at JLab)
- formed **Lattice Hadron Physics Collaboration** (LHPC) in 2000
- acquired funding through DOE SciDAC to build computing cluster at JLab, Fermilab, Brookhaven, develop software
- LHPC has several broad goals
  - compute QCD spectrum (baryons, mesons,...)
  - hadron structure (form factors, structure functions,...)
  - hadron-hadron interactions
- current members of spectroscopy effort:
  - Robert Edwards, George Fleming, Jimmy Juge, Adam Lichtl, CM, Nilmani Mathur, David Richards, Ikuro Sato, Steve Wallace

# LHPC spectroscopy efforts

- extracting spectrum of resonances is big challenge!!
  - need sets of extended operators (correlator matrices)
  - multi-hadron operators needed too
  - deduce resonances from finite-box energies
  - anisotropic lattices ( $a_t < a_s$ )
  - inclusion of light-quark loops at realistically light quark mass
- long-term project
- efforts divide into two categories
  - operator technology
  - Monte Carlo updating technology (light quark loops)
- this talk is an interim status report
  - focus on baryons

# Outline

- how to extract excited-state energies from Monte Carlo computations
  - unstable states (resonances)
- operator construction
  - spatially-extended operators
  - symmetry channels
- field smearing
- operator pruning
- milestone reached: extraction of nine or more levels in a symmetry channel!!
- outlook and conclusion

# Excited states, resonances in lattice Monte Carlo

# Energies from correlation functions

- stationary-state energies extracted from temporal correlations of the fields (in imaginary time formalism)
- evolution in Heisenberg picture  $\phi(t) = e^{Ht} \phi(0) e^{-Ht}$   
(  $H =$  Hamiltonian)
- spectral representation of a simple correlation function

- assume transfer matrix, ignore temporal boundary conditions, focus only on one time ordering

$$\begin{aligned}\langle 0 | \phi(t) \phi(0) | 0 \rangle &= \sum_n \langle 0 | e^{Ht} \phi(0) e^{-Ht} | n \rangle \langle n | \phi(0) | 0 \rangle \quad \leftarrow \text{insert complete set of discrete energy eigenstates} \\ &= \sum_n |\langle n | \phi(0) | 0 \rangle|^2 e^{-(E_n - E_0)t} = \sum_n A_n e^{-(E_n - E_0)t}\end{aligned}$$

- extract  $A_1$  and  $E_1 - E_0$  as  $t \rightarrow \infty$   
(assuming  $\langle 0 | \phi(0) | 0 \rangle = 0$  and  $\langle 1 | \phi(0) | 0 \rangle \neq 0$  )

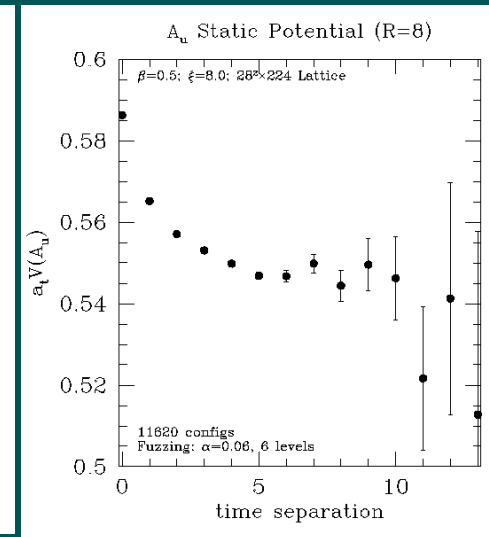
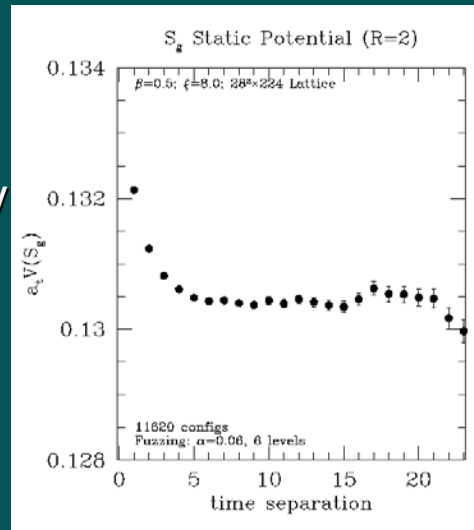
# Effective mass

- the "effective mass" is given by  $m_{\text{eff}}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$
- notice that (take  $E_0 = 0$ )  

$$\lim_{t \rightarrow \infty} m_{\text{eff}}(t) = \ln\left(\frac{A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + \dots}{A_1 e^{-E_1(t+1)} + \dots}\right) \rightarrow \ln e^{E_1} = E_1$$
- effective mass tends to **actual mass** (energy) asymptotically
- effective mass plot is convenient visual tool to **see** signal extraction

□ seen as a **plateau**

- plateau sets in quickly for good operator
- excited-state contamination** before plateau



# Principal correlators

- extracting excited-state energies described in
  - C. Michael, NPB **259**, 58 (1985)
  - Luscher and Wolff, NPB **339**, 222 (1990)
- exploits the variational method
- for  $N \times N$  correlator matrix  $C_{\alpha\beta}(t) = \langle 0 | O_\alpha(t) O_\beta^\dagger(0) | 0 \rangle$  define  $N$  *principal correlators*  $\lambda_\alpha(t, t_0)$  as eigenvalues of

$$C(t_0)^{-1/2} C(t) C(t_0)^{-1/2}$$

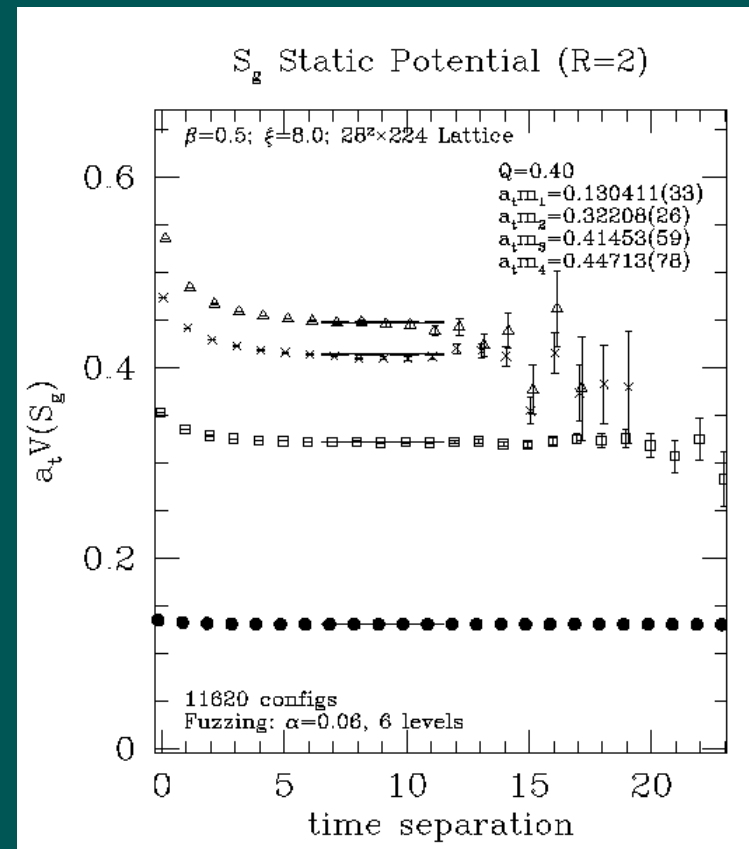
where  $t_0$  (the time defining the "metric") is small

- can show  $\lim_{t \rightarrow \infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha} (1 + e^{-t\Delta E_\alpha})$
- $N$  *principal effective masses*  $\Omega_\alpha(t) = \ln \left( \frac{\lambda_\alpha(t, t_0)}{\lambda_\alpha(t+1, t_0)} \right)$   
now tend (plateau) to  $N$  lowest-lying stationary-state energies



# Principal effective masses

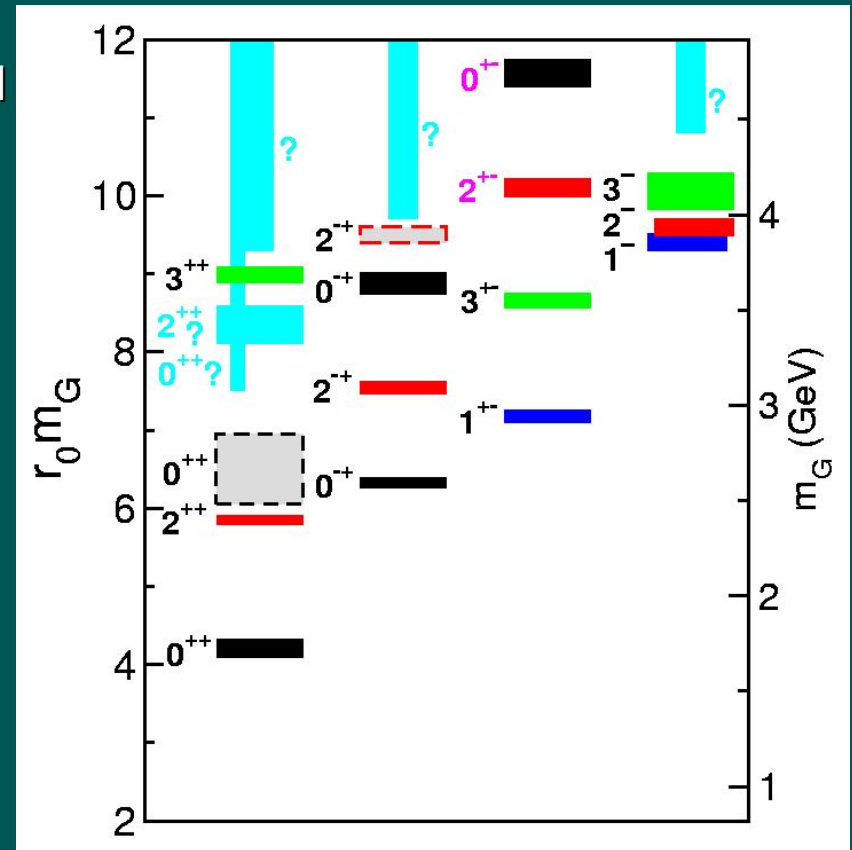
- perform single-exponential fit to each principal correlator to extract spectrum
  - use sum of two-exponentials to minimize sensitivity to  $t_{\min}$
- principal effective masses can cross, approach asymptotic behavior from below
- final results independent of  $t_0$ , choosing larger values of reference time can introduce larger errors



# Yang-Mills SU(3) Glueball Spectrum

- glueball mass spectrum
  - improve scalar states
- mass *ratios* predicted, overall scale is not
- mass gap with \$1 million bounty (Clay mathematics institute)
- glueball structure
  - constituent gluons vs flux loops?

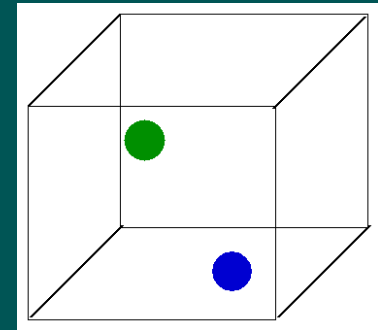
C. Morningstar and M. Peardon,  
Phys. Rev. D 60, 034509 (1999)



$r_0^{-1} = 410(20)$  MeV, states labeled by  $J^{PC}$

# Unstable particles (resonances)

- our computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states → single hadron, 2 hadron, ...
- scattering phase shifts → resonance masses, widths (in principle) deduced from finite-box spectrum
  - B. DeWitt, PR **103**, 1565 (1956) (sphere)
  - M. Luscher, NPB**364**, 237 (1991) (cube)
- modest goal: “ferret” out resonances from scattering states
  - must differentiate resonances from multi-hadron states
  - avoided level crossings, different volume dependences
  - know masses of decay products → placement and pattern of multi-particle states known
  - resonances show up as extra states with little volume dependence



# Resonance in a toy model (I)

- O(4) non-linear  $\sigma$  model (Zimmerman et al, NPB(PS) **30**, 879 (1993))

$$S = -2\kappa \sum_x \sum_{\mu=1}^4 \Phi_a(x) \Phi_a(x + \hat{\mu}) + J \sum_x \Phi^4(x), \quad \sum_{a=1}^4 \Phi_a^2(x) = 1$$

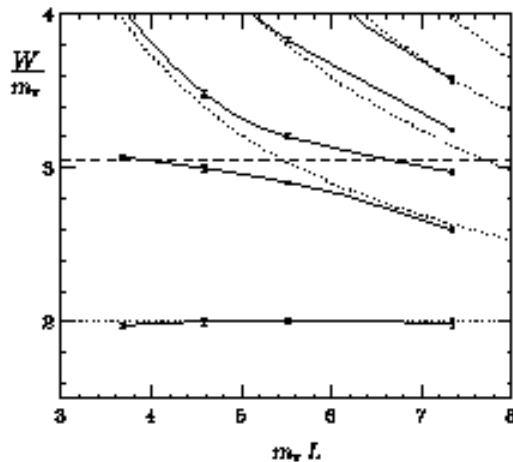


Figure 2. Two-particle energy spectrum

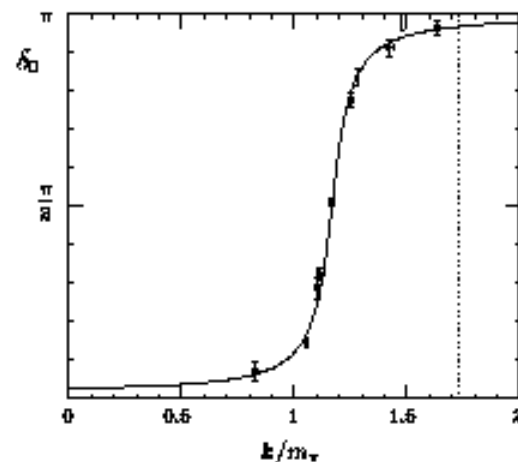


Figure 3. Scattering phase shift  $\delta_0$  in the isospin 0 channel

# Resonance in a toy model (II)

- coupled scalar fields: (Rummukainen and Gottlieb, NPB450, 397 (1995))

$$S = \frac{1}{2} \int d^4x \left( (\partial_\mu \phi)^2 + m_\pi^2 \phi^2 + \lambda \phi^4 + (\partial_\mu \rho)^2 + m_\pi^2 \rho^2 + \lambda_\rho \rho^4 + g \rho \phi^2 \right)$$

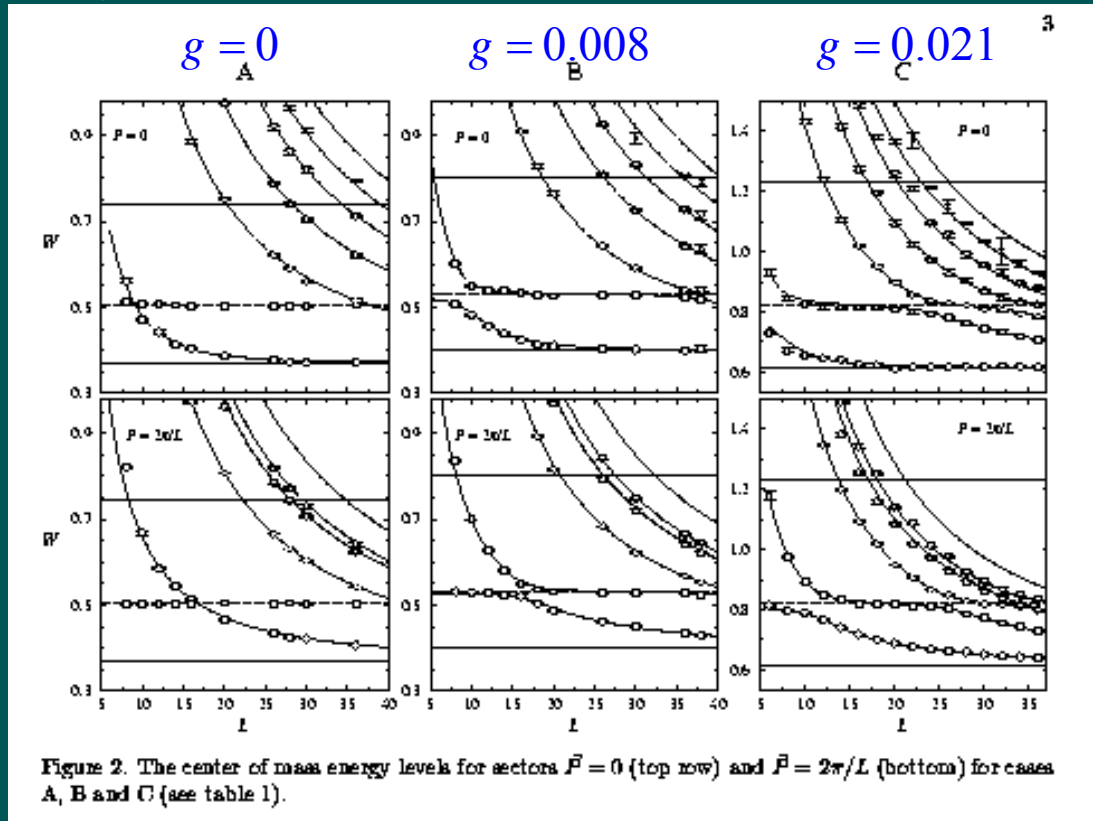


Figure 2. The center of mass energy levels for sectors  $\vec{P} = 0$  (top row) and  $\vec{P} = 2\pi/L$  (bottom) for cases A, B and C (see table 1).

# Operator construction

# Operator design issues

- must facilitate spin identification
  - shun the usual method of operator construction which relies on cumbersome continuum space-time constructions
  - focus on constructing operators which transform irreducibly under the symmetries of the lattice
- one eye on maximizing overlaps with states of interest
- other eye on minimizing number of quark-propagator sources
- use building blocks useful for baryons, mesons, multi-hadron operators
- must project onto definite spin polarizations or will observe many degeneracies

# Three stage approach

- concentrate on baryons at rest (zero momentum)
- operators classified according to the irreps of  $O_h$

$$G_{1g}, G_{1u}, G_{2g}, G_{2u}, H_g, H_u$$

- (1) basic building blocks: smeared, covariant-displaced quark fields  $(\tilde{D}_j^{(p)}\tilde{\psi}(x))_{Aa\alpha}$   $p$ -link displacement ( $j = 0, \pm 1, \pm 2, \pm 3$ )

- (2) construct **elemental** operators (translationally invariant)

$$B^F(x) = \phi_{ABC}^F \varepsilon_{abc} (\tilde{D}_i^{(p)}\tilde{\psi}(x))_{Aa\alpha} (\tilde{D}_j^{(p)}\tilde{\psi}(x))_{Bb\beta} (\tilde{D}_k^{(p)}\tilde{\psi}(x))_{Cc\gamma}$$

- flavor structure from isospin, color structure from gauge invariance
- (3) group-theoretical projections onto irreps of  $O_h$

$$B_i^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R \in O_h^D} D_{\lambda\lambda}^{(\Lambda)}(R)^* U_R B_i^F(t) U_R^+$$

- Grassmann package in Maple to do these calculations
- details in [PRD72, 094506 \(2005\)](#)



# Three-quark elemental operators

- three-quark operator

$$\Phi_{\alpha\beta\gamma,ijk}^{ABC}(t) = \sum_{\vec{x}} \varepsilon_{abc} (\tilde{D}_i^{(p)} \tilde{\psi}(\vec{x}, t))_{a\alpha}^A (\tilde{D}_j^{(p)} \tilde{\psi}(\vec{x}, t))_{b\beta}^B (\tilde{D}_k^{(p)} \tilde{\psi}(\vec{x}, t))_{c\gamma}^C$$

- covariant displacements

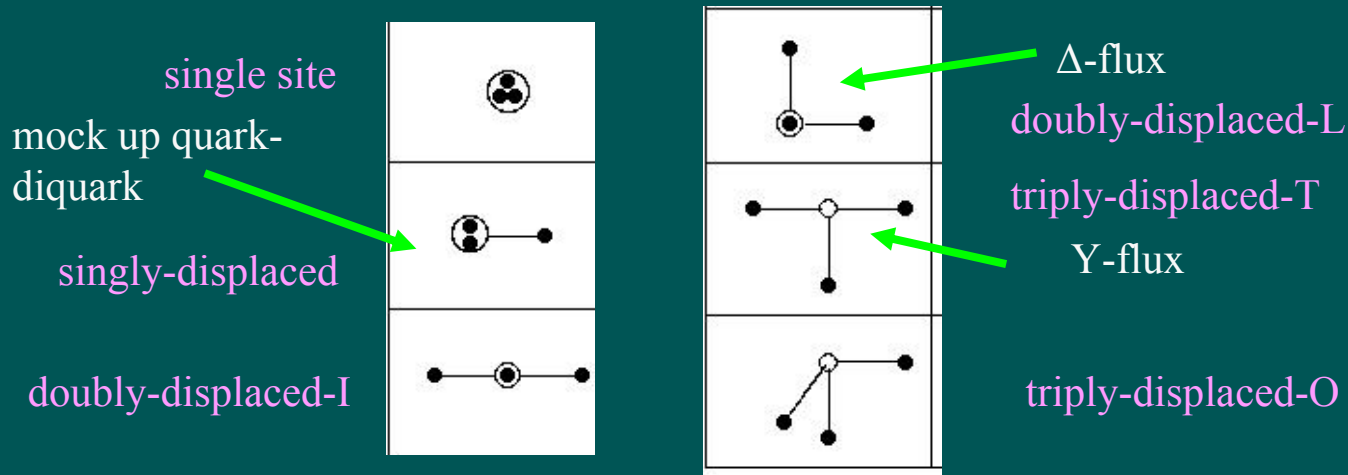
$$\tilde{D}_j^{(p)}(x, x') = \tilde{U}_j(x) \tilde{U}_j(x + \hat{j}) \cdots \tilde{U}_j(x + (p-1)\hat{j}) \delta_{x', x+p\hat{j}} \quad (j = \pm 1, \pm 2, \pm 3)$$

$$\tilde{D}_0^{(p)}(x, x') = \delta_{x', x}$$

Baryon	Operator
$\Delta^{++}$	$\Phi_{\alpha\beta\gamma,ijk}^{uuu}$
$\Sigma^+$	$\Phi_{\alpha\beta\gamma,ijk}^{uus}$
$N^+$	$\Phi_{\alpha\beta\gamma,ijk}^{uud} - \Phi_{\alpha\beta\gamma,ijk}^{duu}$
$\Xi^0$	$\Phi_{\alpha\beta\gamma,ijk}^{ssu}$
$\Lambda^0$	$\Phi_{\alpha\beta\gamma,ijk}^{uds} - \Phi_{\alpha\beta\gamma,ijk}^{dus}$
$\Omega^-$	$\Phi_{\alpha\beta\gamma,ijk}^{sss}$

# Incorporating orbital and radial structure

- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure



- minimizes number of sources for quark propagators
- useful for mesons, tetraquarks, pentaquarks even!
- can even incorporate **hybrid mesons** operator (in progress)

# Enumerating the three-quark operators

- lots of operators (too many!)

	$\Delta^{++}, \Omega^{-}$	$\Sigma^{+}, \Xi^{0}$	$N^{+}$	$\Lambda^{0}$
Single-site	20	40	20	24
Singly-displaced	240	624	384	528
Doubly-displaced-I	192	572	384	576
Doubly-displaced-L	768	2304	1536	2304
Triply-displaced-T	768	2304	1536	2304
Triply-displaced-O	512	1536	1024	1536

# Spin identification and other remarks

- spin identification possible by pattern matching

$J$	$n_{G_1}^J$	$n_{G_2}^J$	$n_H^J$
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3

total numbers of operators assuming two different displacement lengths

Irrep	$\Delta, \Omega$	$N$	$\Sigma, \Xi$	$\Lambda$
$G_{1g}$	221	443	664	656
$G_{1u}$	221	443	664	656
$G_{2g}$	188	376	564	556
$G_{2u}$	188	376	564	556
$H_g$	418	809	1227	1209
$H_u$	418	809	1227	1209

- total numbers of operators is huge  $\rightarrow$  uncharted territory
- ultimately must face two-hadron scattering states

# Single-site operators

- choose Dirac-Pauli convention for  $\gamma$ -matrices

- 20 independent single-site  $\Delta^{++}$  elemental operators:

$$\Delta_{\alpha\beta\gamma} = \epsilon_{abc} \bar{u}_{a\alpha} \bar{u}_{b\beta} \bar{u}_{c\gamma}, \quad (\alpha \leq \beta \leq \gamma)$$

- 20 independent single-site  $N^+$  elemental operators:

$$N_{\alpha\beta\gamma} = \epsilon^{abc} (\bar{u}_{a\alpha} \bar{u}_{b\beta} \bar{d}_{c\gamma} - \bar{d}_{a\alpha} \bar{u}_{b\beta} \bar{u}_{c\gamma}), \quad (\alpha \leq \beta, \alpha < \gamma)$$

- 40 independent single-site  $\Sigma^+$  elemental operators:

$$\Sigma_{\alpha\beta\gamma} = \epsilon_{abc} \bar{u}_{a\alpha} \bar{u}_{b\beta} \bar{s}_{c\gamma} \quad (\alpha \leq \beta)$$

- 24 independent single-site  $\Lambda^0$  elemental operators:

$$\Lambda_{\alpha\beta\gamma} = \epsilon_{abc} (\bar{u}_{a\alpha} \bar{d}_{b\beta} \bar{s}_{c\gamma} - \bar{d}_{a\alpha} \bar{u}_{b\beta} \bar{s}_{c\gamma}) \quad (\alpha < \beta)$$

# $\Delta^{++}$ single-site operators

Irrep	Row	DP Operators
$G_{1g}$	1	$\Delta_{144} - \Delta_{234}$
$G_{1g}$	2	$-\Delta_{134} + \Delta_{233}$
$G_{1u}$	1	$\Delta_{124} - \Delta_{223}$
$G_{1u}$	2	$-\Delta_{114} + \Delta_{123}$
$H_g$	1	$\Delta_{222}$
$H_g$	2	$-\sqrt{3} \Delta_{122}$
$H_g$	3	$\sqrt{3} \Delta_{112}$
$H_g$	4	$-\Delta_{111}$
$H_g$	1	$\sqrt{3} \Delta_{244}$
$H_g$	2	$-\Delta_{144} - 2\Delta_{234}$
$H_g$	3	$2\Delta_{134} + \Delta_{233}$
$H_g$	4	$-\sqrt{3} \Delta_{133}$

Irrep	Row	DP Operators
$H_u$	1	$\sqrt{3} \Delta_{224}$
$H_u$	2	$-2\Delta_{124} - \Delta_{223}$
$H_u$	3	$\Delta_{114} + 2\Delta_{123}$
$H_u$	4	$-\sqrt{3} \Delta_{113}$
$H_u$	1	$\Delta_{444}$
$H_u$	2	$-\sqrt{3} \Delta_{344}$
$H_u$	3	$\sqrt{3} \Delta_{334}$
$H_u$	4	$-\Delta_{333}$

# Single-site $N_+$ operators

Irrep	Row	DP Operators
$G_{1g}$	1	$N_{122}$
$G_{1g}$	2	$-N_{112}$
$G_{1g}$	1	$N_{144} - N_{243}$
$G_{1g}$	2	$-N_{134} + N_{233}$
$G_{1g}$	1	$N_{144} - 2N_{234} + N_{243}$
$G_{1g}$	2	$N_{134} - 2N_{143} + N_{233}$
$G_{1u}$	1	$N_{142}$
$G_{1u}$	2	$-N_{132}$
$G_{1u}$	1	$N_{344}$
$G_{1u}$	2	$-N_{334}$
$G_{1u}$	1	$2N_{124} - N_{142} - 2N_{223}$
$G_{1u}$	2	$-2N_{114} + 2N_{123} - N_{132}$

Irrep	Row	DP Operators
$H_g$	1	$\sqrt{3} N_{244}$
$H_g$	2	$-N_{144} - N_{234} - N_{243}$
$H_g$	3	$N_{134} + N_{143} + N_{233}$
$H_g$	4	$-\sqrt{3} N_{133}$
$H_u$	1	$\sqrt{3} N_{224}$
$H_u$	2	$-2N_{124} + N_{142} - N_{223}$
$H_u$	3	$N_{114} + 2N_{123} - N_{132}$
$H_u$	4	$-\sqrt{3} N_{113}$

# Quark-field and link-variable smearing issues



# Run parameters

- run parameters for all results presented here
  - $12^3 \times 48$  anisotropic lattice
  - Wilson gauge, Wilson fermion actions
  - lattice spacings  $a_s \sim 0.1 \text{ fm}$ ,  $a_s / a_t \sim 3.0$
  - quark masses such that  $m_\pi \sim 700 \text{ MeV}$
  - quenched
  - correlator matrices averaged over irrep rows
  - use of opposite-parity time-reversed propagators to double statistics
  - number of configurations used
    - 50 for operator smearing tests
    - 200 for operator prunings

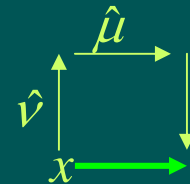
# Quark- and gauge-field smearing

- smeared quark and gluon fields → dramatically reduced coupling with short wavelength modes

- **link-variable** smearing (stout links PRD**69**, 054501 (2004))

- define  $C_\mu(x) = \sum_{\pm(v \neq \mu)} \rho_{\mu\nu} U_\nu(x) U_\mu(x + \hat{\nu}) U_\nu^\dagger(x + \hat{\mu})$

- spatially isotropic  $\rho_{jk} = \rho, \quad \rho_{4k} = \rho_{k4} = 0$



- exponentiate traceless Hermitian matrix

$$\Omega_\mu = C_\mu U_\mu^+ \quad Q_\mu = \frac{i}{2} (\Omega_\mu^+ - \Omega_\mu) - \frac{i}{2N} \text{Tr}(\Omega_\mu^+ - \Omega_\mu)$$

- iterate

$$U_\mu^{(n+1)} = \exp(iQ_\mu^{(n)}) U_\mu^{(n)}$$

$$U_\mu \rightarrow U_\mu^{(1)} \rightarrow \dots \rightarrow U_\mu^{(n)} \equiv \tilde{U}_\mu$$

- **quark**-field smearing (covariant Laplacian uses smeared gauge field)

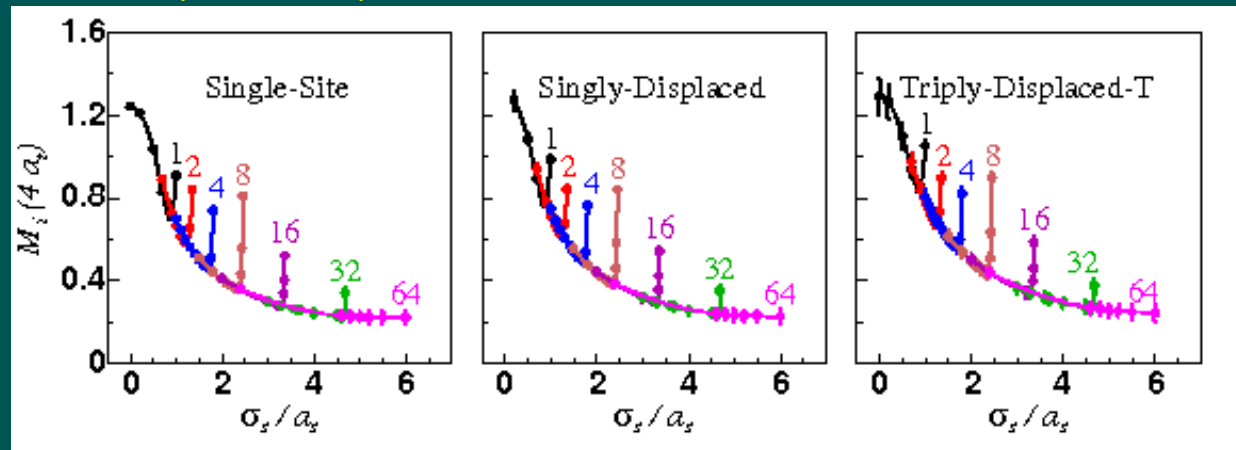
$$\tilde{\psi}(x) = \left( 1 + \frac{\sigma_s^2}{4n_\sigma} \tilde{\Delta}^2 \right)^{n_\sigma} \psi(x)$$

- parameters to tune:  $\sigma_s, n_\sigma, \rho, n_\rho$

# Quark-field smearing tuning

- focus on three particular operators for smearing tests
  - a single-site operator  $O_{SS}$  in the  $G_{1g}$  irrep
  - a singly-displaced operator  $O_{SD}$  with a particular choice of Dirac indices and 3-link displacement length
  - a triply-displaced-T operator  $O_{TDT}$  with a particular choice of the Dirac indices (3-link displacement lengths)
- define effective mass as usual  $M_i(t) = \ln\left(\frac{C_{ii}(t)}{C_{ii}(t+a_t)}\right)$
- use  $M_i(t=4a_t)$  to compare different quark-field smearings
- smeared links  $\rho n_\rho = 2.5, n_\rho = 16$  since displaced operators noisy

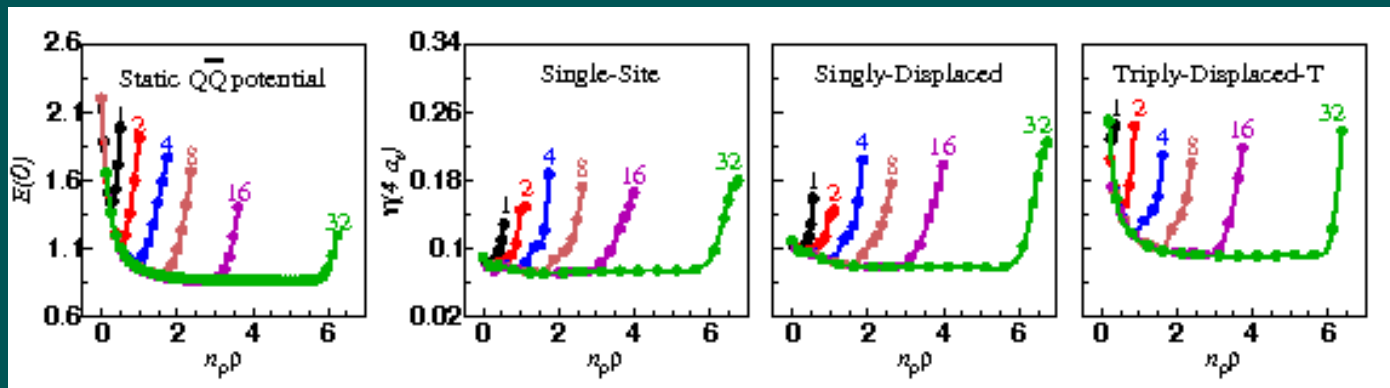
$$n_\sigma = 1, 2, 4, 8, \\ 16, 32, 64$$



# Link-variable smearing tuning

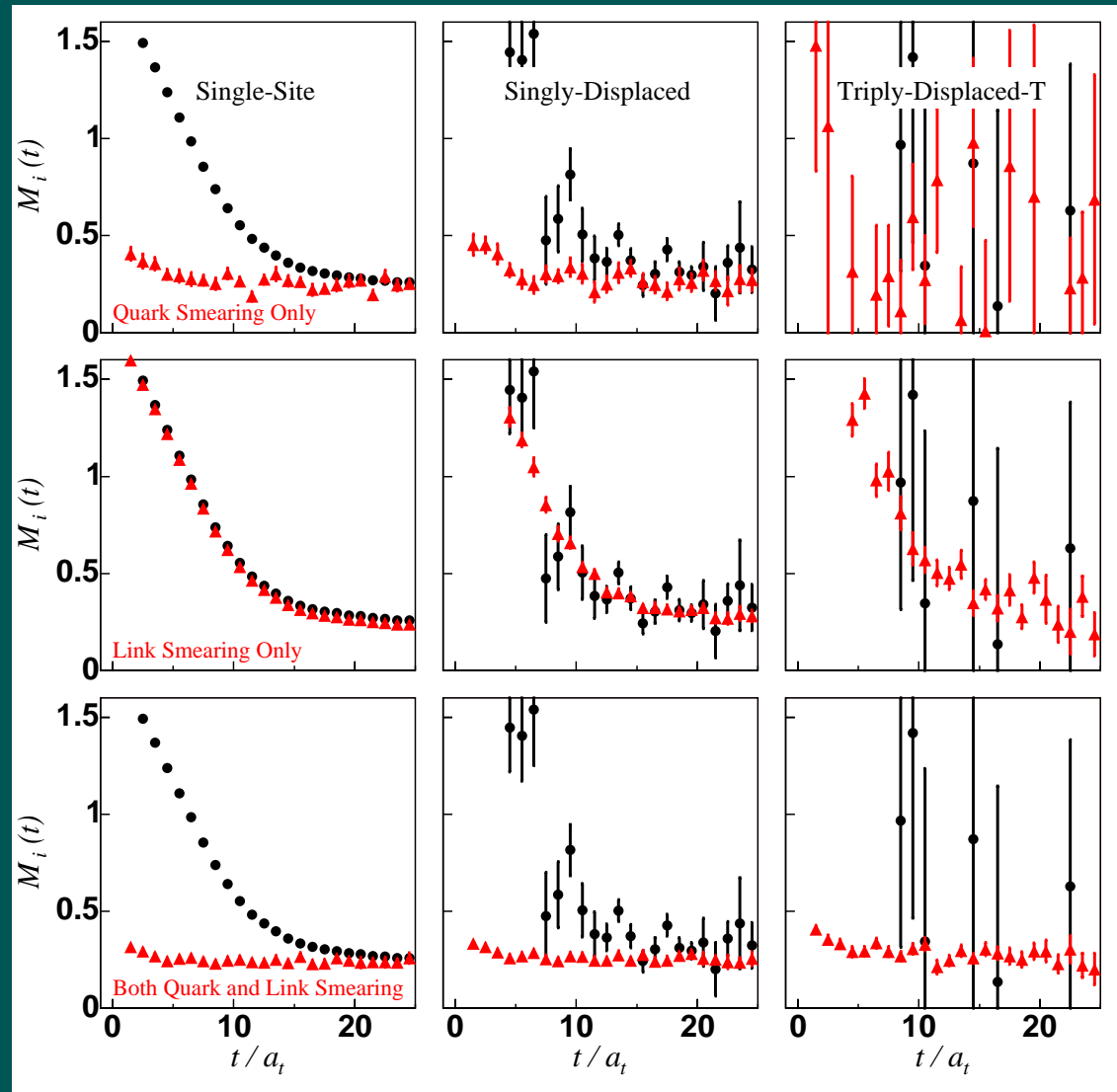
- first, used the effective mass  $E(t=0)$  associated with the static quark-antiquark potential at spatial separation  $R = 5a_s \sim 0.5 \text{ fm}$
- found that link-smearing did not appreciably alter values of baryon effective masses, but had dramatic effect on *variance*
- compared relative jackknife error  $\eta_i(t=4a_t)$  of  $M_i(t=4a_t)$  for different link-smearing parameters ( $\sigma_s = 4.0, n_\sigma = 32$ )
- lesson learned: preferred parameters from static potential produce smallest errors in baryon effective masses

$$n_\rho = 1, 2, 4, 8, 16, 32$$



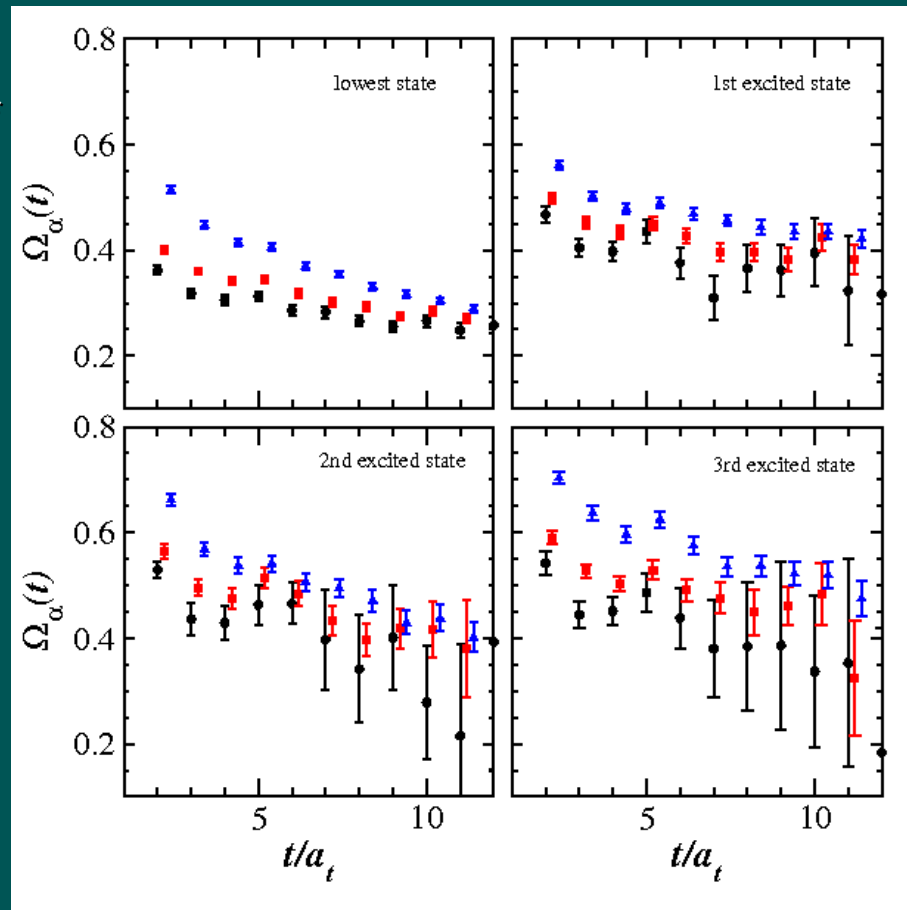
# Importance of smearing

- Nucleon G1g channel
- effective masses of the 3 selected operators
- noise reduction from link variable smearing, especially for displaced operators
- quark-field smearing reduces couplings to high-lying states
  - $\sigma_s = 4.0, n_\sigma = 32$
  - $n_\rho \rho = 2.5, n_\rho = 16$
- effect on excited states shows  $\sigma_s = 3.0$  better



# Smearing and excited states

- previous tests involved lowest state only
- important to tune smearing for excited states as well
- lowest 4 principal effective masses for 10x10 matrix of DDI  $G_{1g}$  operators shown
  - same link smearing  
 $\rho n_\rho = 2.5, n_\rho = 16$
  - quark-field smearings  
 $\sigma_s = 2.0$  (blue),  $3.0$  (red),  
 $4.0$  (black)  $n_\sigma = 32$
  - $\rho n_\rho = 5.0, n_\rho = 32$   
with  $\sigma_s = 4.0$  did not  
reduce errors
- preferred:  $n_\sigma = 32, \sigma_s = 3.0$



# Smearing summary

- From our quenched study of the  $G_{1g}$  nucleon channel on small lattices  $12^3 \times 48$  for  $a_s \sim 0.1 \text{ fm}$  and  $a_s / a_t \sim 3.0$  and  $m_\pi \sim 700 \text{ MeV}$ , the preferred smearing parameters are

$$\rho n_\rho = 2.5, n_\rho = 16$$

$$n_\sigma = 32, \sigma_s = 3.0$$

- factors still to consider:
  - evidence for same smearing for other irreps
  - expect same smearing for other isospin channels
  - dependence on lattice spacing
  - dependence on quark mass

# Operator pruning issues



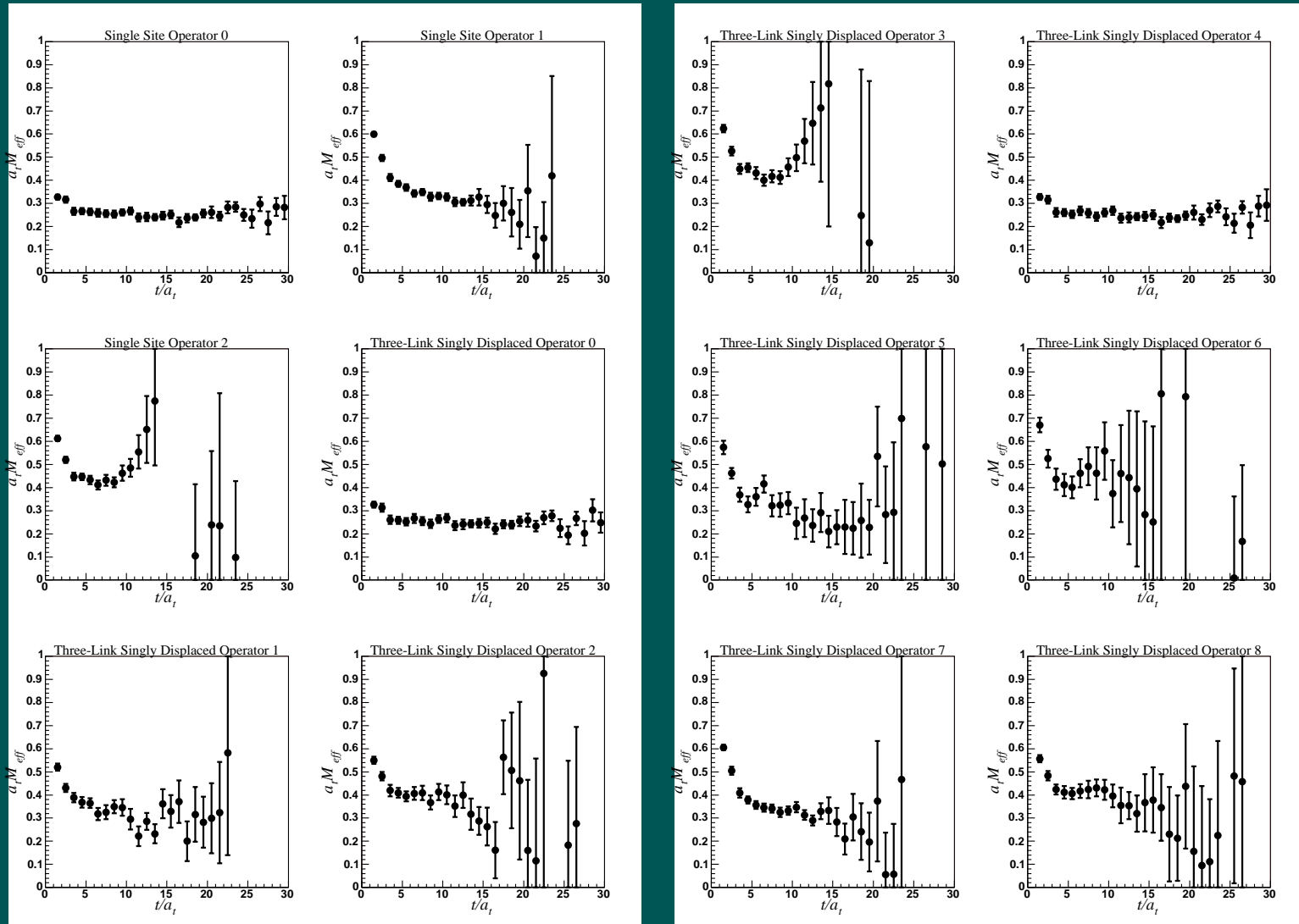
# Operator plethora

- Number of  $N^+$  operators given below (1 displacement length)
  - total of 179 operators in  $G_{1g}$  channel

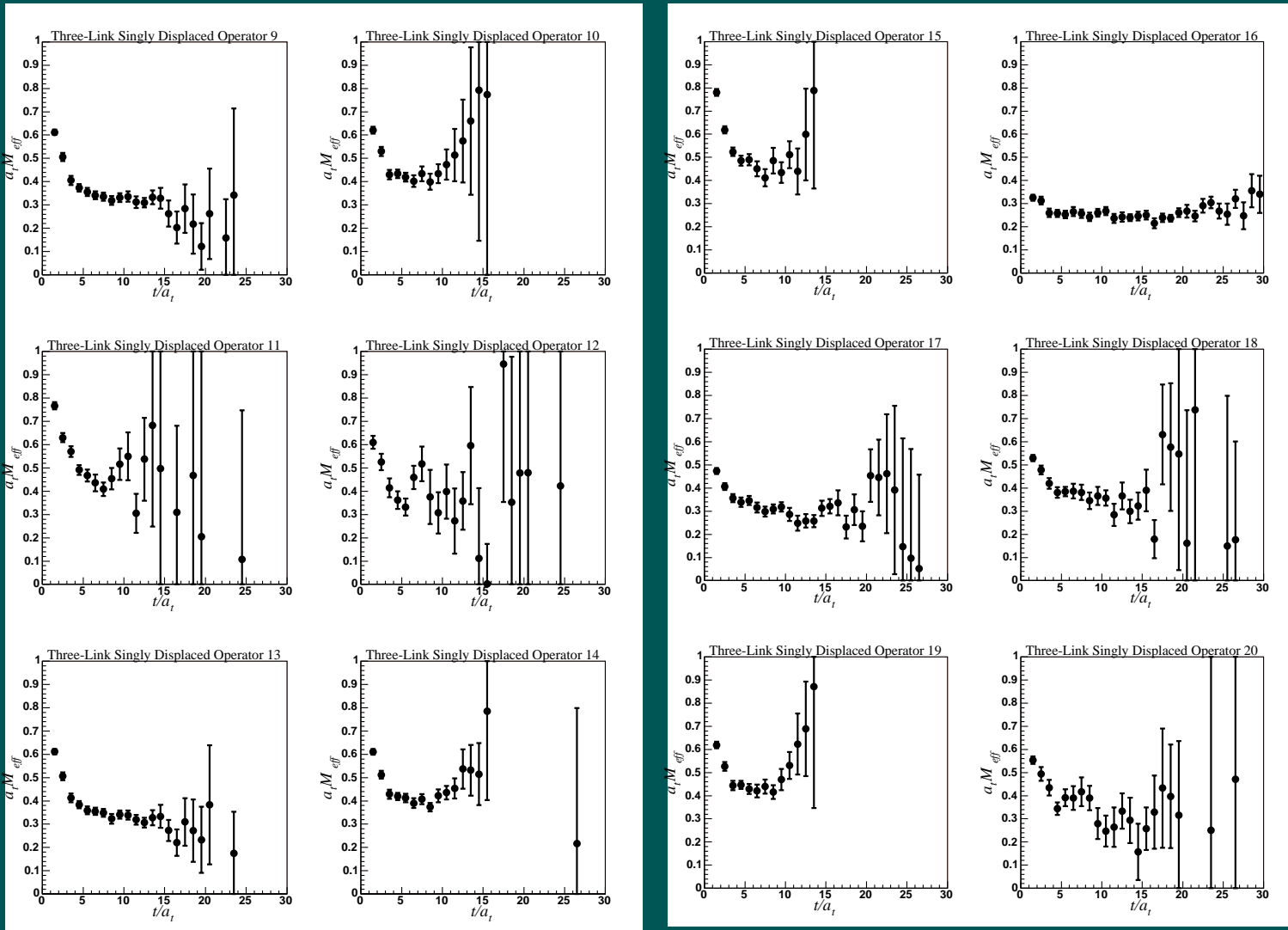
	$G_{1g}$	$G_{2g}$	$H_g$
Single-site	3	0	1
Singly-displaced	24	8	32
Doubly-displaced-I	24	8	32
Doubly-displaced-L	64	64	128
Triply-displaced-T	64	64	128

- since 179x179 matrix too large to be practical, operator pruning is clearly necessary
- will focus on  $G_{1g}$  channel first

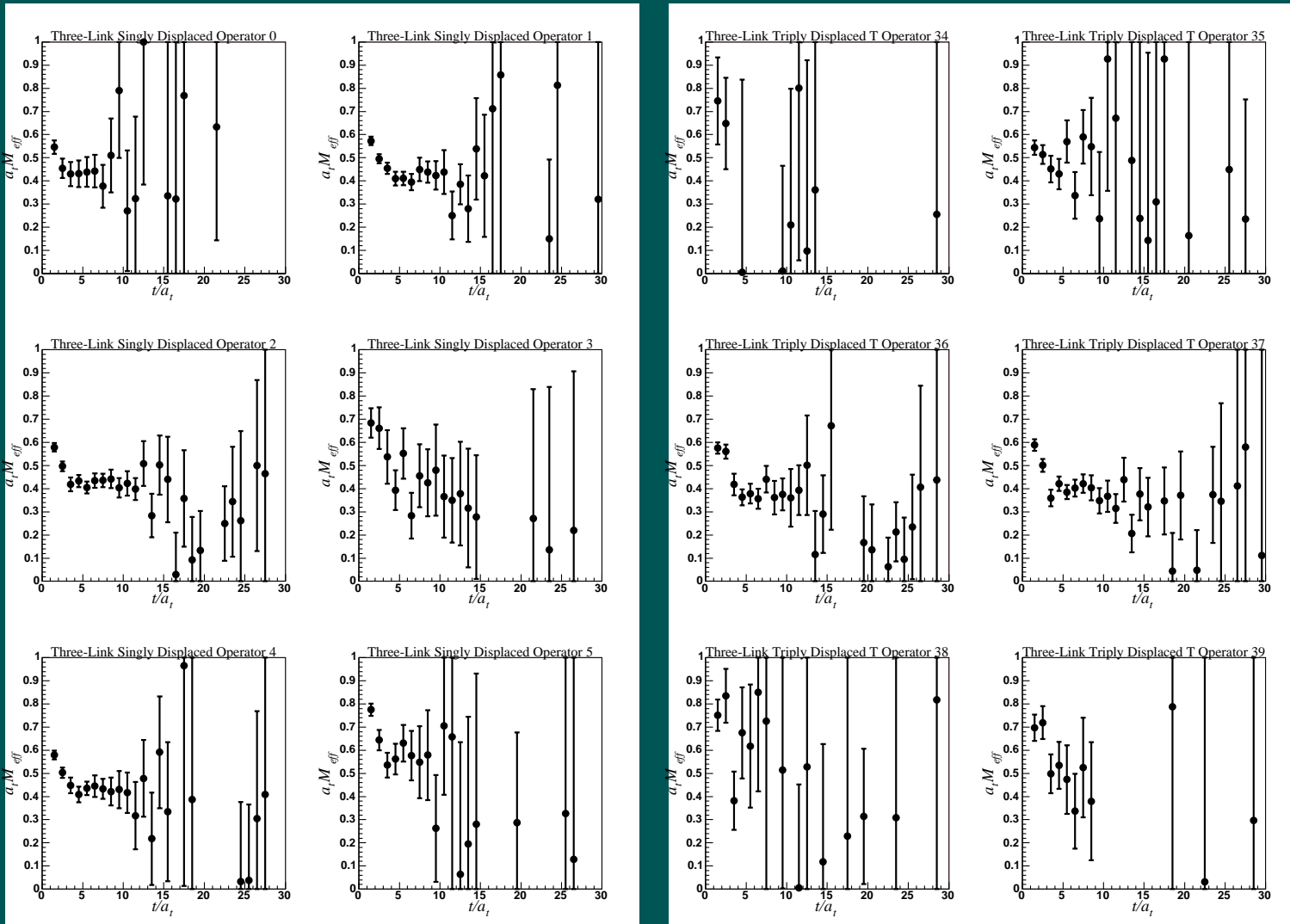
# Operator plethora (G1g Nucleon)



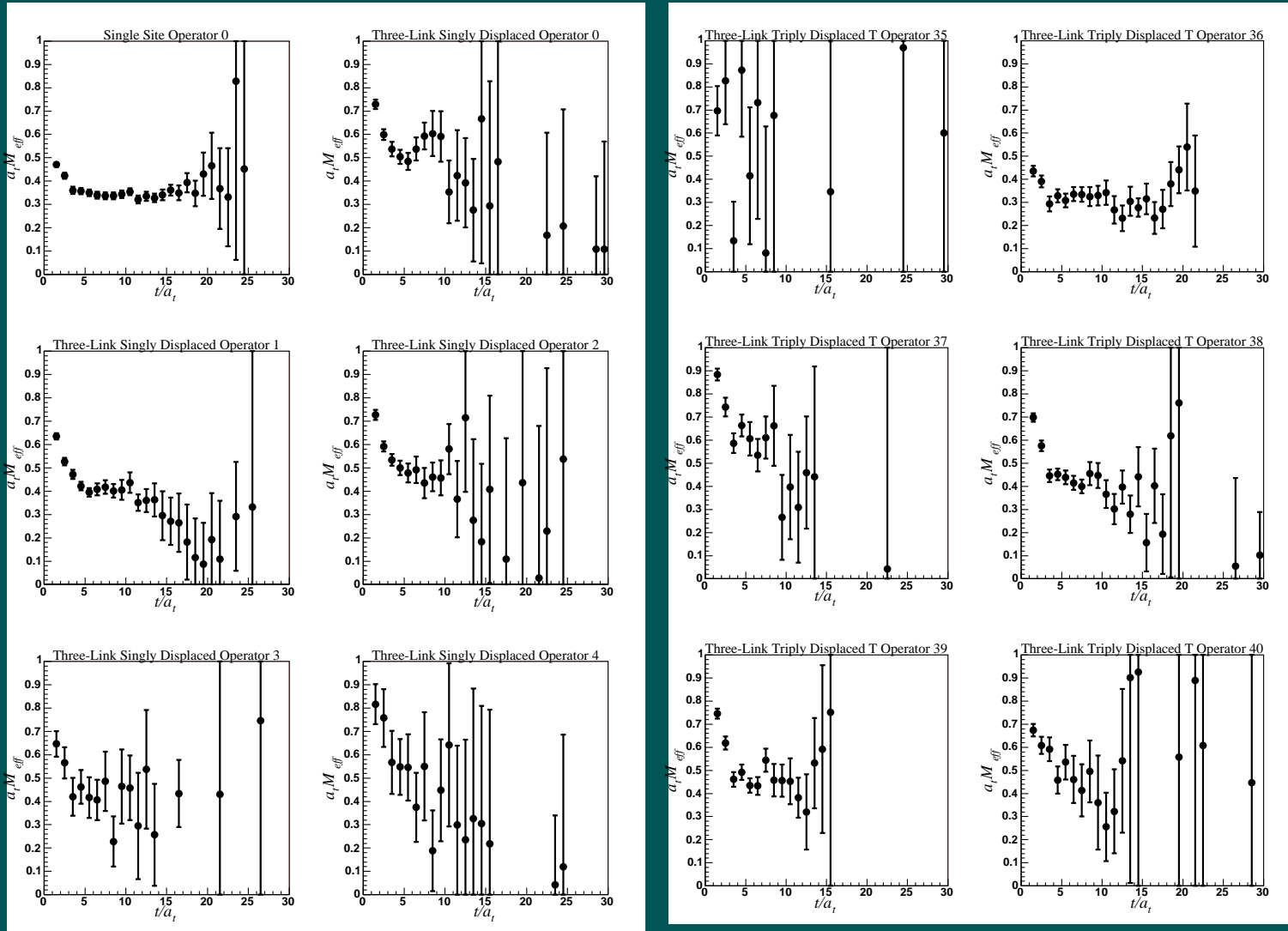
# G1g nucleon operators



# G2g nucleon operators

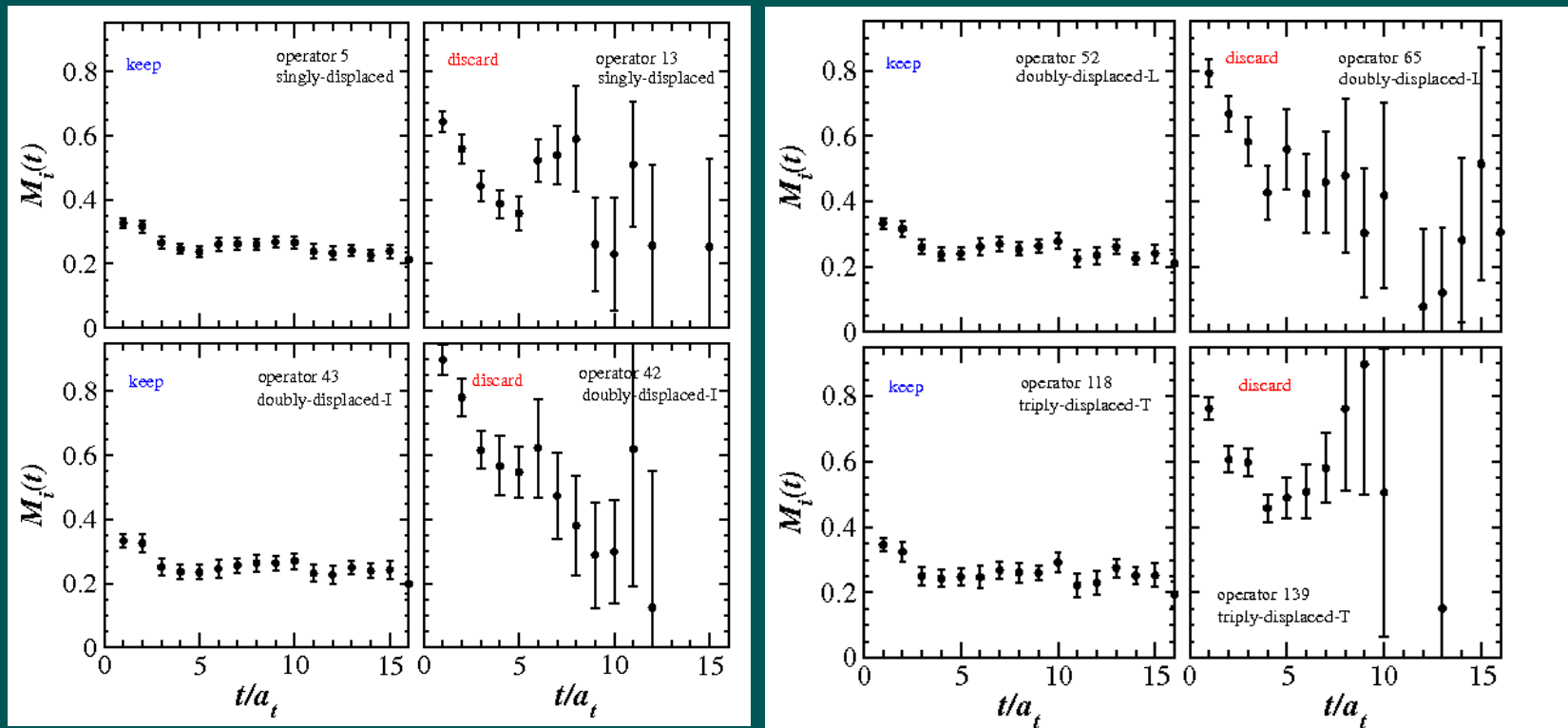


# Hu nucleon operators



# Pruning: step one

- look at effective masses of diagonal elements of correlation matrix and discard noisy operators
- examples shown below ( $\rho n_\rho = 2.5, n_\rho = 16, n_\sigma = 32, \sigma_s = 4.0$  used)
- all 179 effective masses



# Pruning: step one (continued)

- retain 64 operators out of the 179
  - SS: 0, 1, 2
  - SD: 3, 5, 6, 8, 10, 12, 14, 17, 19, 20, 22, 24, 25
  - DDI: 27, 29, 30, 31, 33, 36, 38, 41, 43, 44, 45, 48
  - DDL: 52, 54, 56, 57, 58, 59, 60, 61, 62, 72, 74, 76, 78, 85, 88, 94, 97, 98, 105, 110
  - TDT: 116, 118, 119, 124, 125, 126, 132, 134, 136, 138, 149, 158, 162, 163, 169, 174

# Noise removal via singular values

- examine renormalized matrix at early time  $t=1$

$$\hat{C}_{ij}(1) = C_{ij}(1) / \sqrt{C_{ii}(1)C_{jj}(1)}$$

- sort operators in order of increasing noise
- include operators in this order, checking singular values
  - skip operators which introduce small singular values
  - presence of small singular values indicates operator set not “independent enough” → noise can creep in

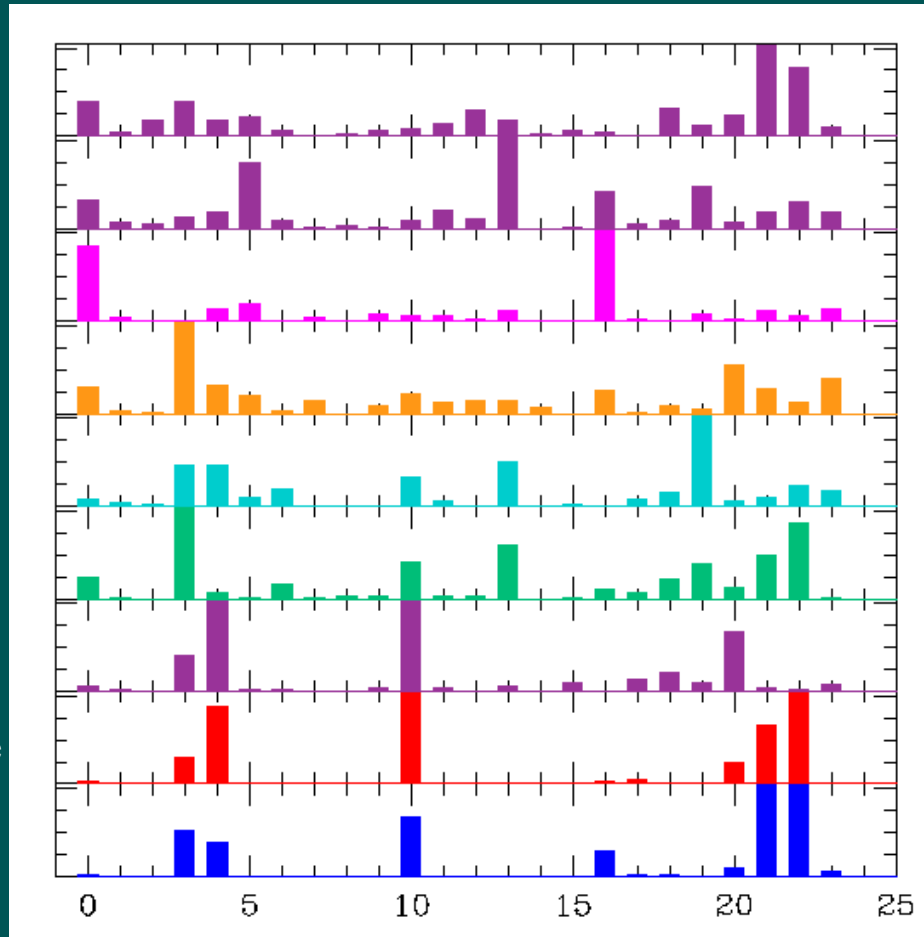


# Further pruning via variational method

- look at eigenvectors of  $C(t_0)^{-1/2} C(t) C(t_0)^{-1/2}$  for  $t=3$
- coefficients squared shown
- retain 12 dominant operators

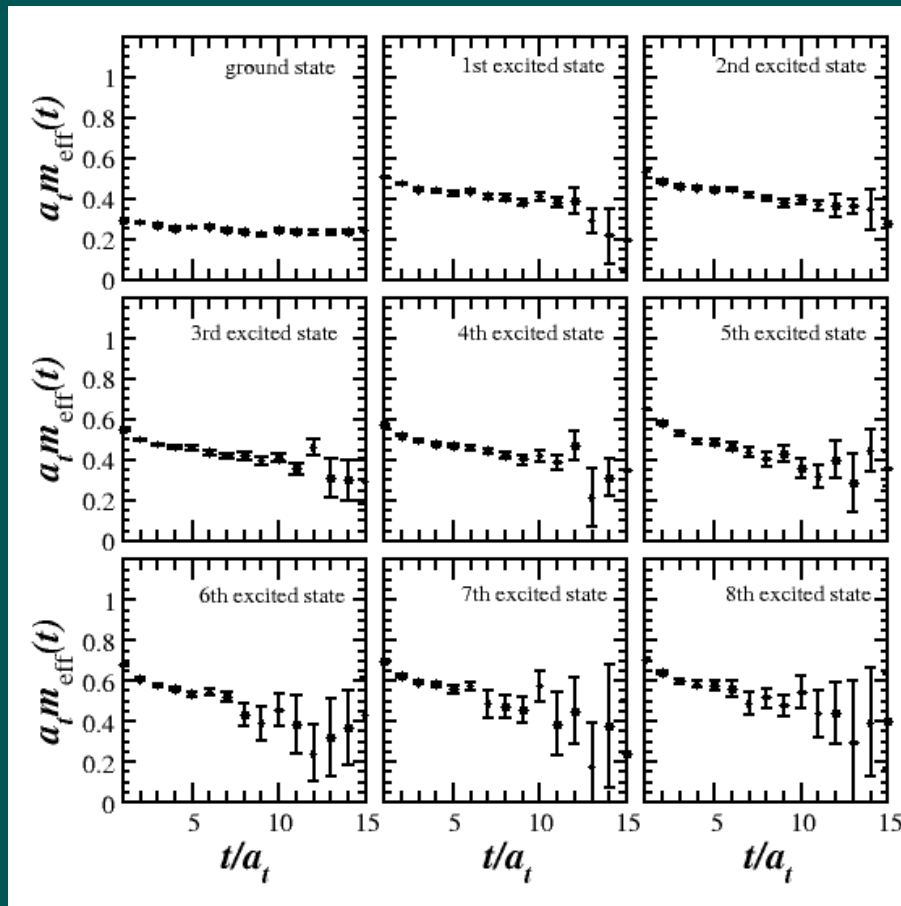
1<sup>st</sup> excited state

ground state

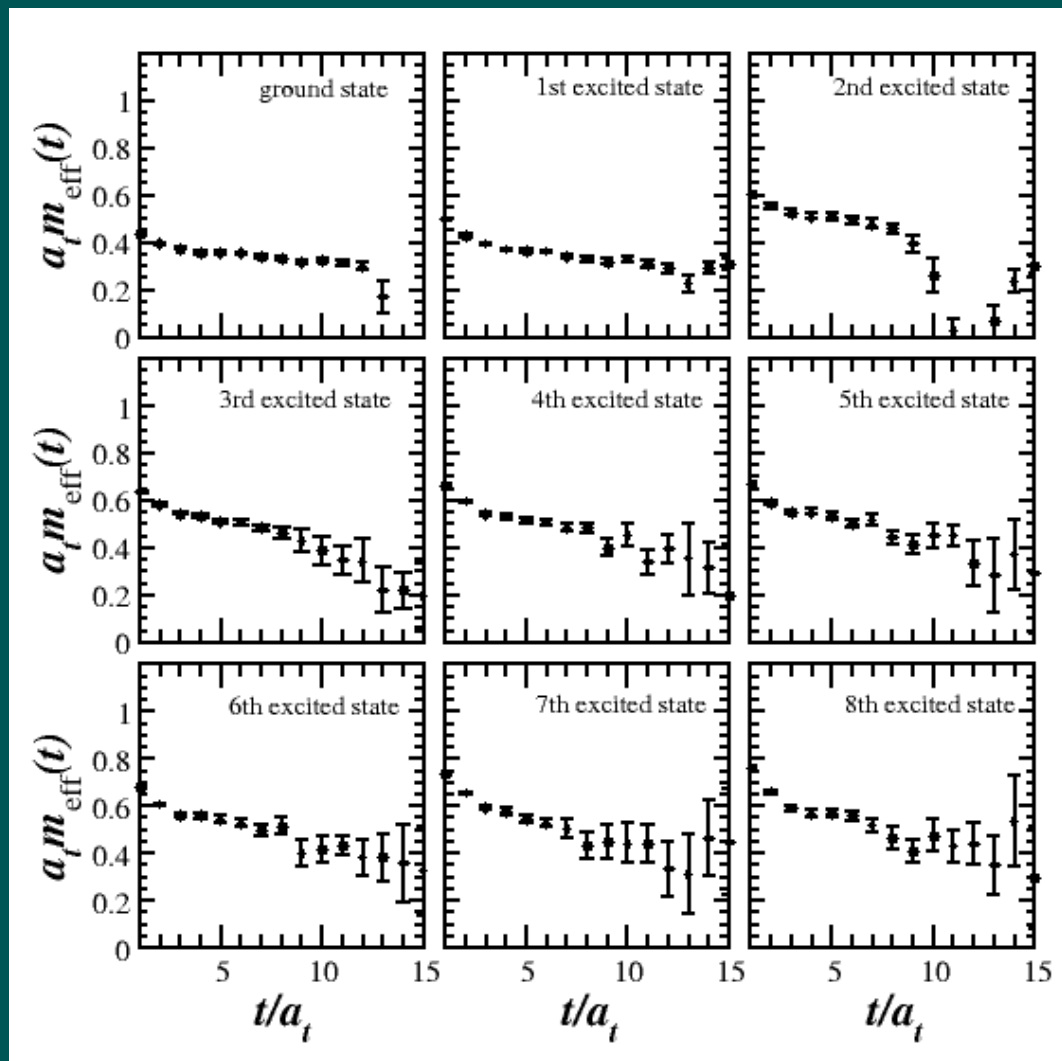


# Milestone: principal effective masses

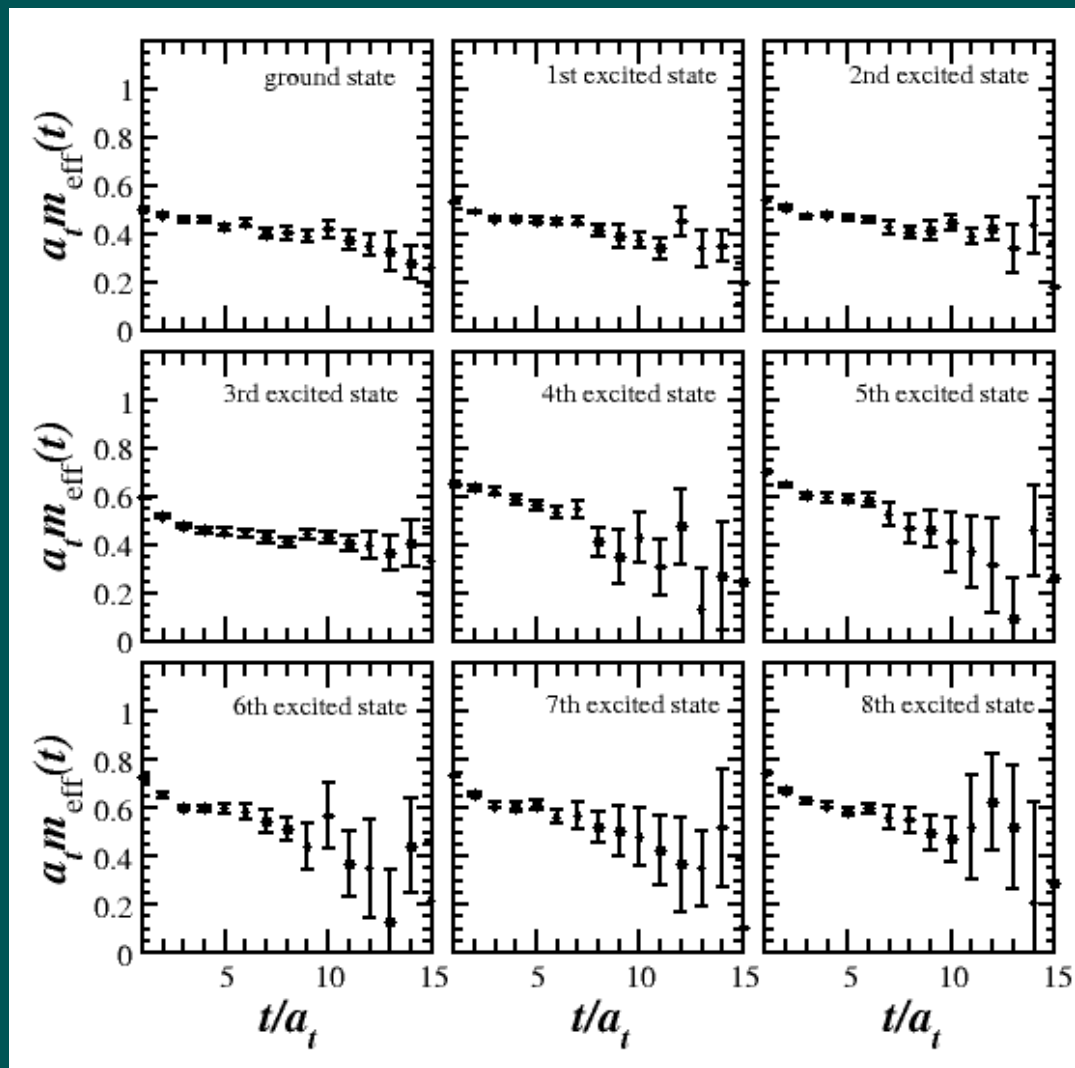
- G1g nucleons (16x16 matrix) using 200 configurations
- world record: 9 levels extracted (even more possible!)



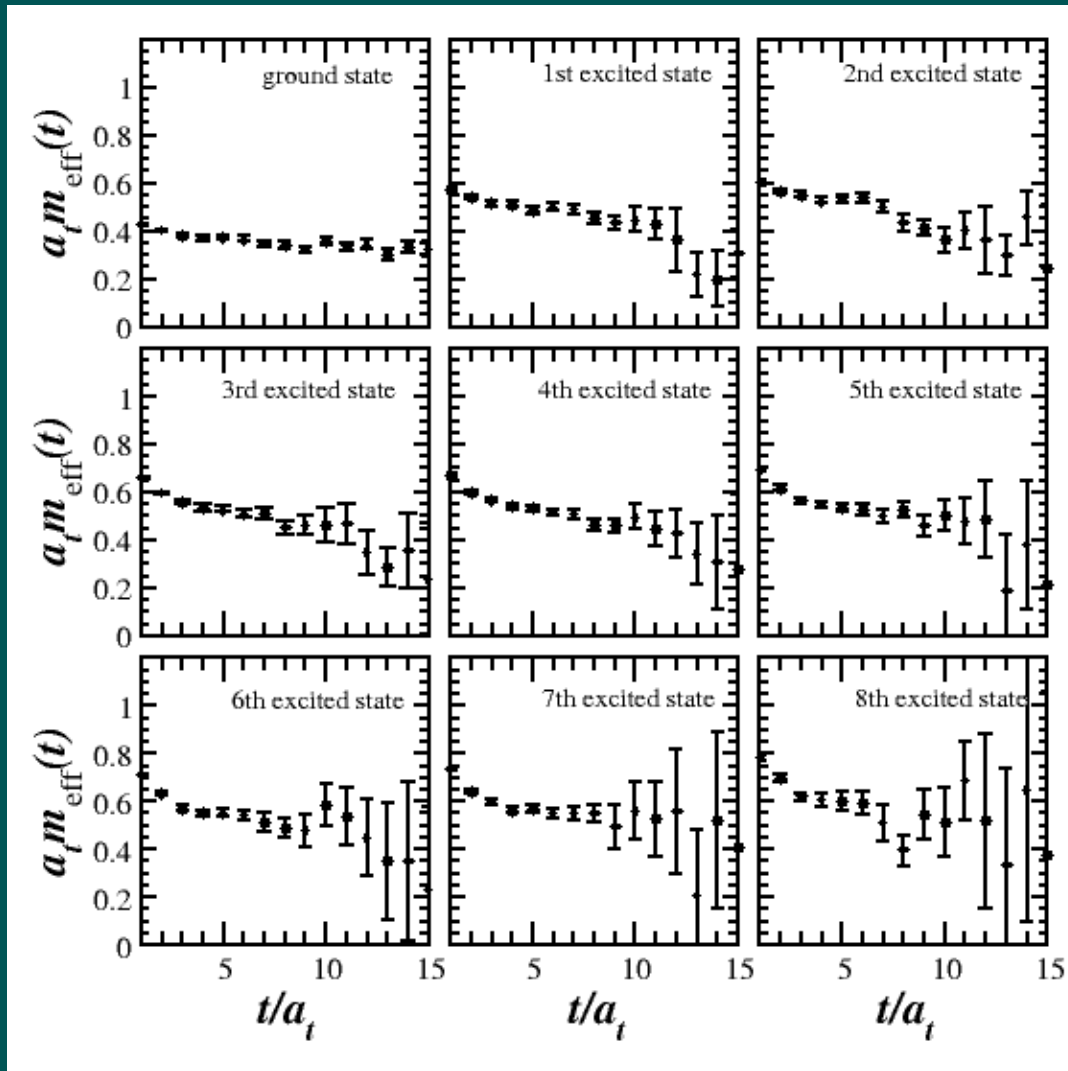
# G1u Nucleon channel



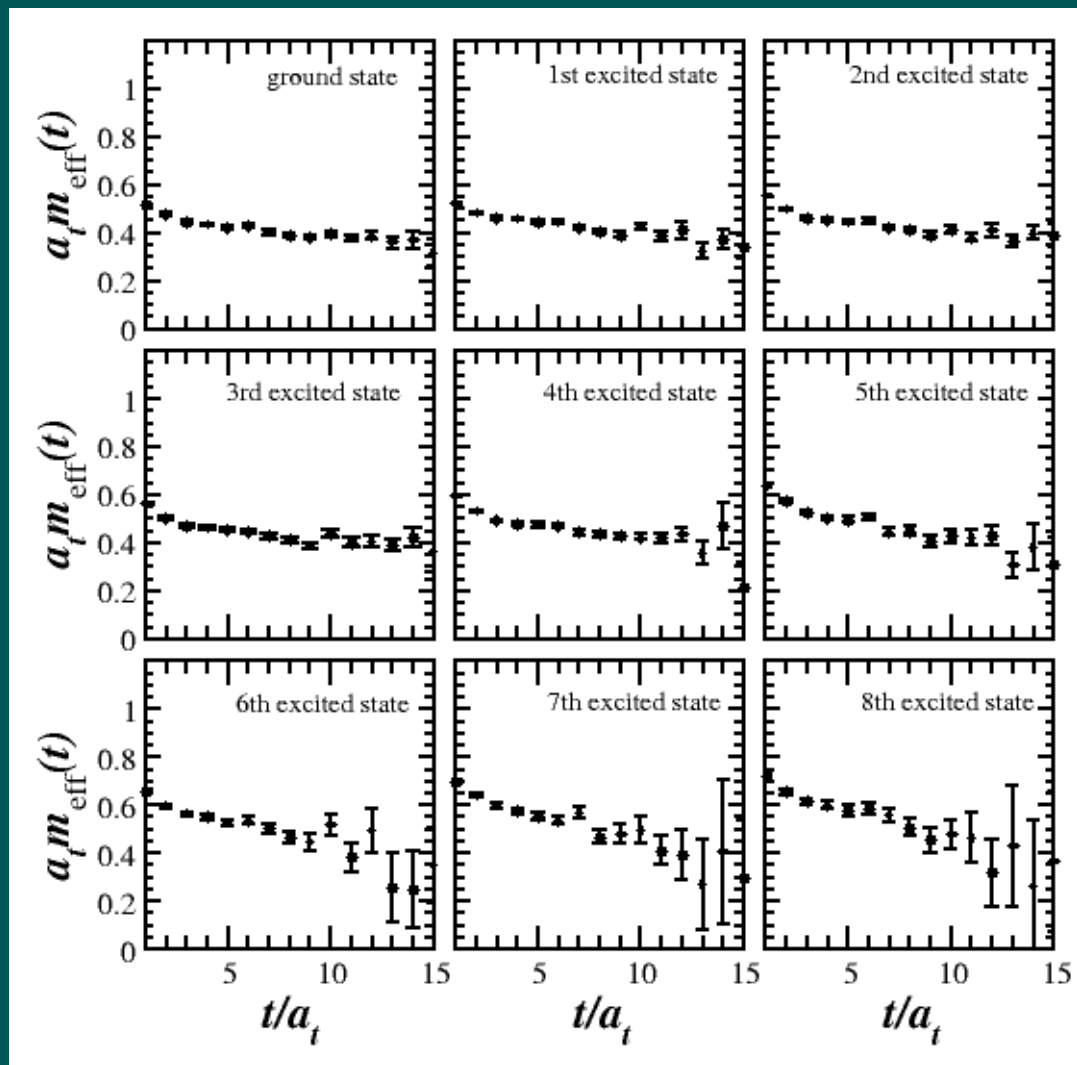
# G2g nucleon channel



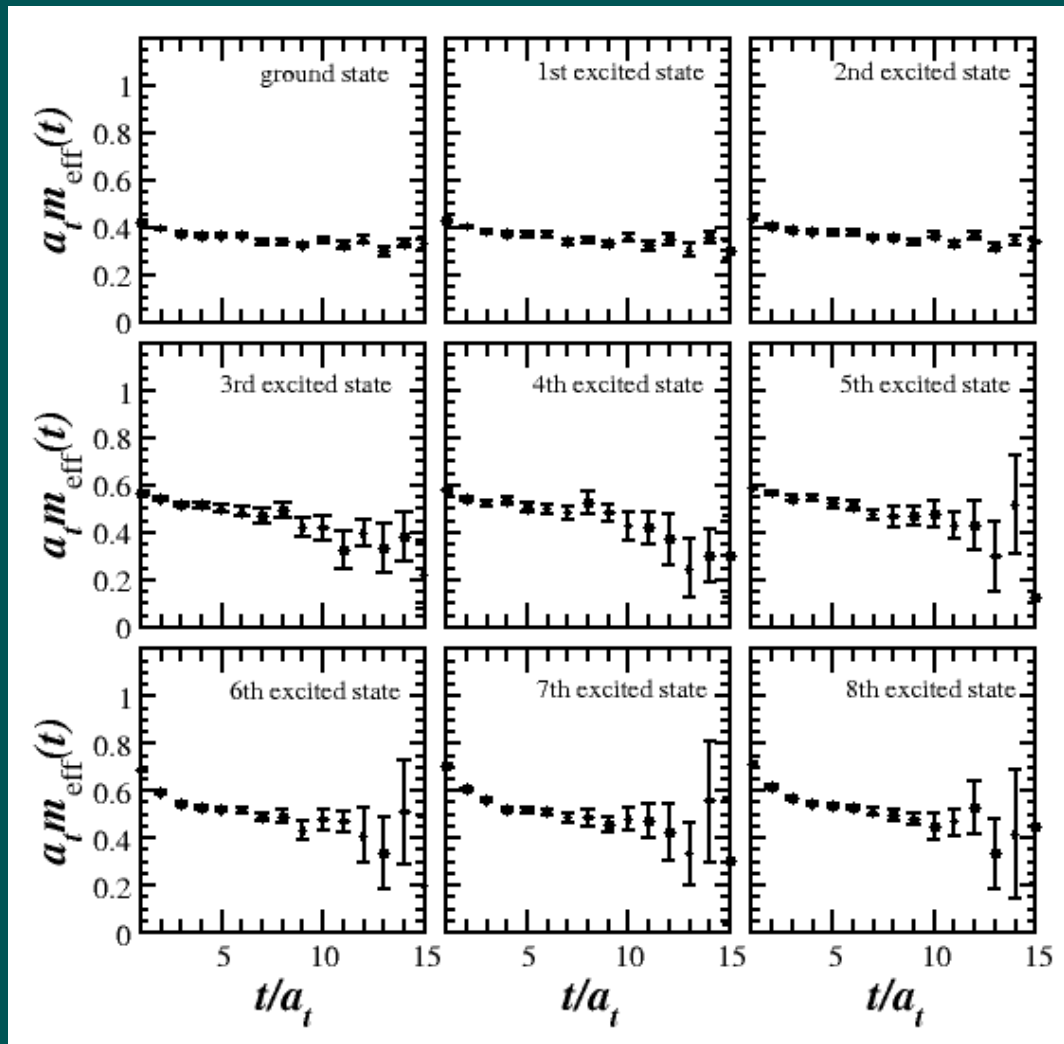
# G2u nucleon channel



# Hg nucleon channel



# Hu nucleon channel



# Future work

- three-quark operator pruning to be completed
  - all irreps, all isospin channels
  - different displacement lengths
- include mesons (in progress)
- include multi-hadron operators
  - stochastic all-to-all propagators with dilution, low eigenvectors
  - will also facilitate disconnected diagrams
- goal: operator technology ready to go when unquenched simulations near realistically-light quark masses become possible
- LHPC recently was awarded 1/6 of QCDOC to generate configurations on large lattices with dynamical quarks such that pion mass around 300 MeV



# Summary

- progress report on ongoing efforts of LHPC to extract hadron spectrum with large sets of extended operators
  - need for correlation matrices of good operators
  - spin identification must be addressed
  - multi-hadron operators will become important
- exploration of 3-quark baryon operators nearly done
- study of meson operators beginning
- major milestone reached: can extract 9 or more states!!
- ultimately, MC updating with realistically light quark masses needed
- very challenging calculations
  - will keep hammering at it!

