ECT* Workshop on
Antiproton Physics & QCD
July 3-7, 2006

Partial-wave analysis of antiproton-proton reactions: Past (LEAR) & future (FAIR)*

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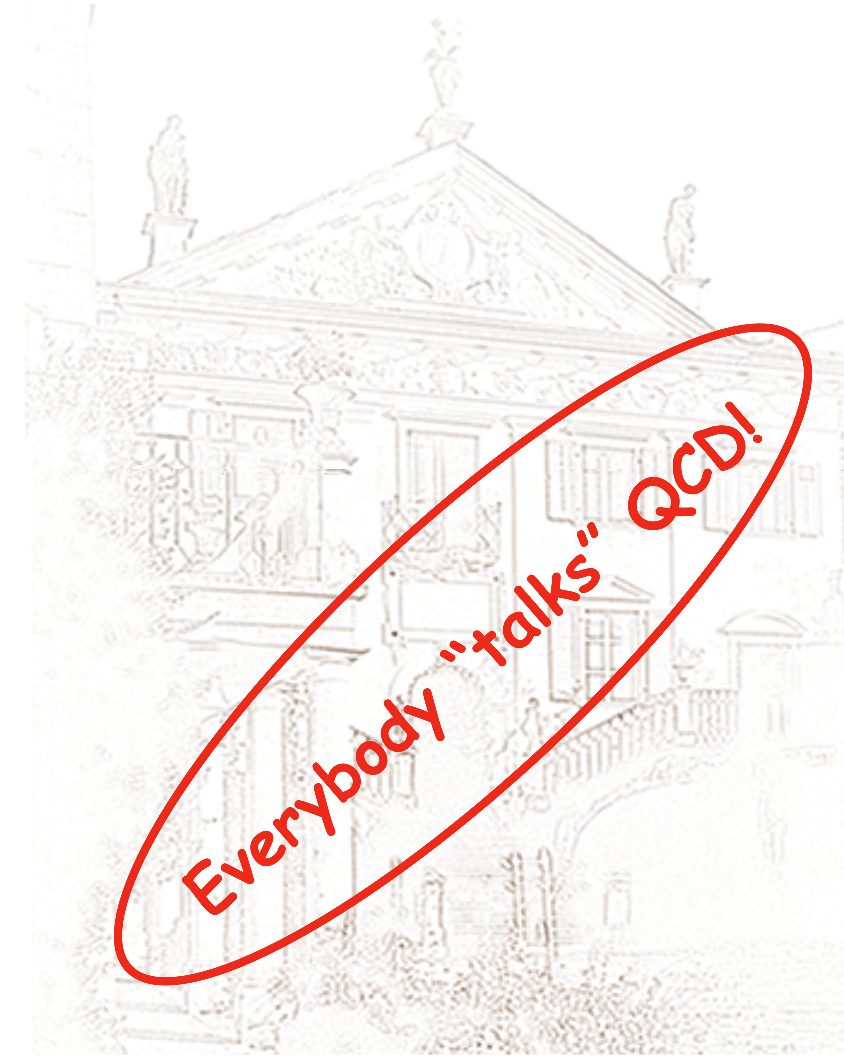
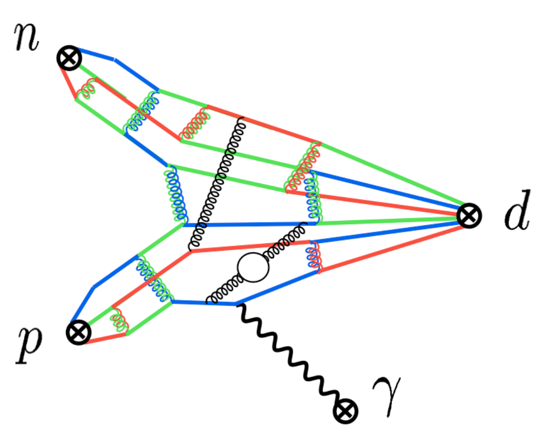
* R.T. *et al.* (1991-1995) & *in preparation.*

ECT* European Centre for Theoretical Studies in Nuclear Physiscs and Related Areas

Nuclear Forces & QCD: Never the Twain Shall Meet*?

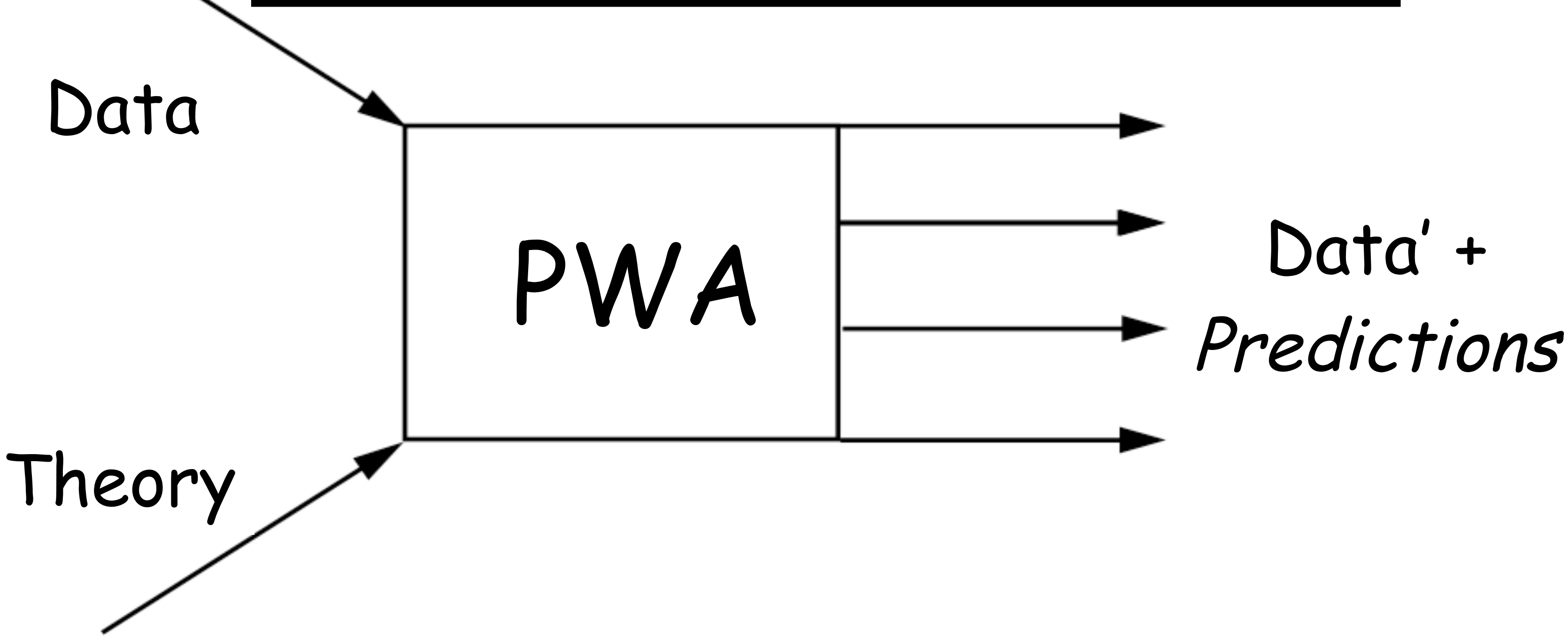


Judith McGovern
Ben Gibson
Bira van Kolck
Rob Timmermans



* Rudyard Kipling: THE BALLAD OF EAST AND WEST (1892)

Experiment ⇔ PWA ⇔ Theory



- Database ≡ *Every* experimental data set (points + errors, statistical & systematic) published in a regular physics journal
- Theory ≡ *As* model independent as possible theory input

PWA is a special expertise in hadronic physics (*e.g.* πN, NN).
It should not be forgotten for antiproton physics at PANDA@HESR
[*Cf.* A major US laboratory below the Mason-Dixie line…]

Experiment ⇔ PWA ⇔ Theory

(Our) PWA is a high-precision tool to:

- improve models for the NN and $\overline{N}$N interaction
- study & improve the database, plan new expt's
- study the long-range interaction V_L
- search for resonances

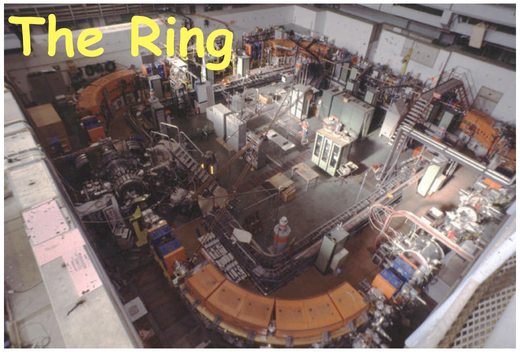
Input into the PWA:

- the "raw" database (~5000 pp, ~5000 np, ~3500 $\overline{N}$N points)
- theory (model independent): EM interaction, OPE + χTPE

Output of the PWA:

- S-matrix: phase-shifts, inelasticities + errors: $\delta_L(E), \eta_L(E)$
- correlations (χ^2–hypersurface)
- parameters in V_L, *e.g.* the pion-nucleon coupling constant
- resonance parameters
- "true" database + rejected data sets

Expt's at LEAR@CERN



Group	Reaction	Observable	p_L (MeV/c)
PS172	$\bar{p}+p\to\bar{p}+p$	σ_{tot}, $d\sigma/d\Omega$, A_y, D_{yy}	200-1500
PS173	$\bar{p}+p\to\bar{p}+p$	σ_{ann} , $d\sigma/d\Omega$	180-600
	$\bar{p}+p\to\bar{n}+n$	σ_{cex} , $d\sigma/d\Omega$	180-600
PS185	$\bar{p}+p\to\bar{Y}+Y$	$d\sigma/d\Omega$, P_y, C_{ij}, D_{nn}, K_{nn}	1435-1900
PS198	$\bar{p}+p\to\bar{p}+p$	$d\sigma/d\Omega$, A_y, D_{yy}	440-700
PS199	$\bar{p}+p\to\bar{n}+n$	$d\sigma/d\Omega$, A_y	600-1300
PS206	$\bar{p}+p\to\bar{n}+n$	$d\sigma/d\Omega$	693

The database of the NN PWA (below 350 MeV)

Proton-proton

Type	# data
σ_{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$	—
$d\sigma/d\Omega$	947
A_y	816
A_{ii}, C_{nn}	876
D, D_t	114
R, R', A, A'	237
Rest	36
All	*3026*

High quality (~15% rejected)

Neutron-proton

Type	# data
σ_{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$	275
$d\sigma/d\Omega$	1475
A_y	1213
A_{yy}, A_{zz}	327
D_t	122
R_t, R_t', A_t, A_t'	162
Rest	78
All	*3652*

Reasonable quality (~25% rejected)

The database of the $\bar{N}N$ PWA (below 925 MeV/c)

Type	LEAR ($\bar{p}+p \rightarrow \bar{p}+p$)	rest ($\bar{p}+p \rightarrow \bar{p}+p$)	LEAR ($\bar{p}+p \rightarrow \bar{n}+n$)	rest ($\bar{p}+p \rightarrow \bar{n}+n$)
σ_{tot}, σ_{ann}	124	–	–	63
$d\sigma/d\Omega$	281	2507	91	154
A_y	200	29	89	–
D_{yy}	5	–	9	–
Total	*610*	*2536*	*189*	*217*

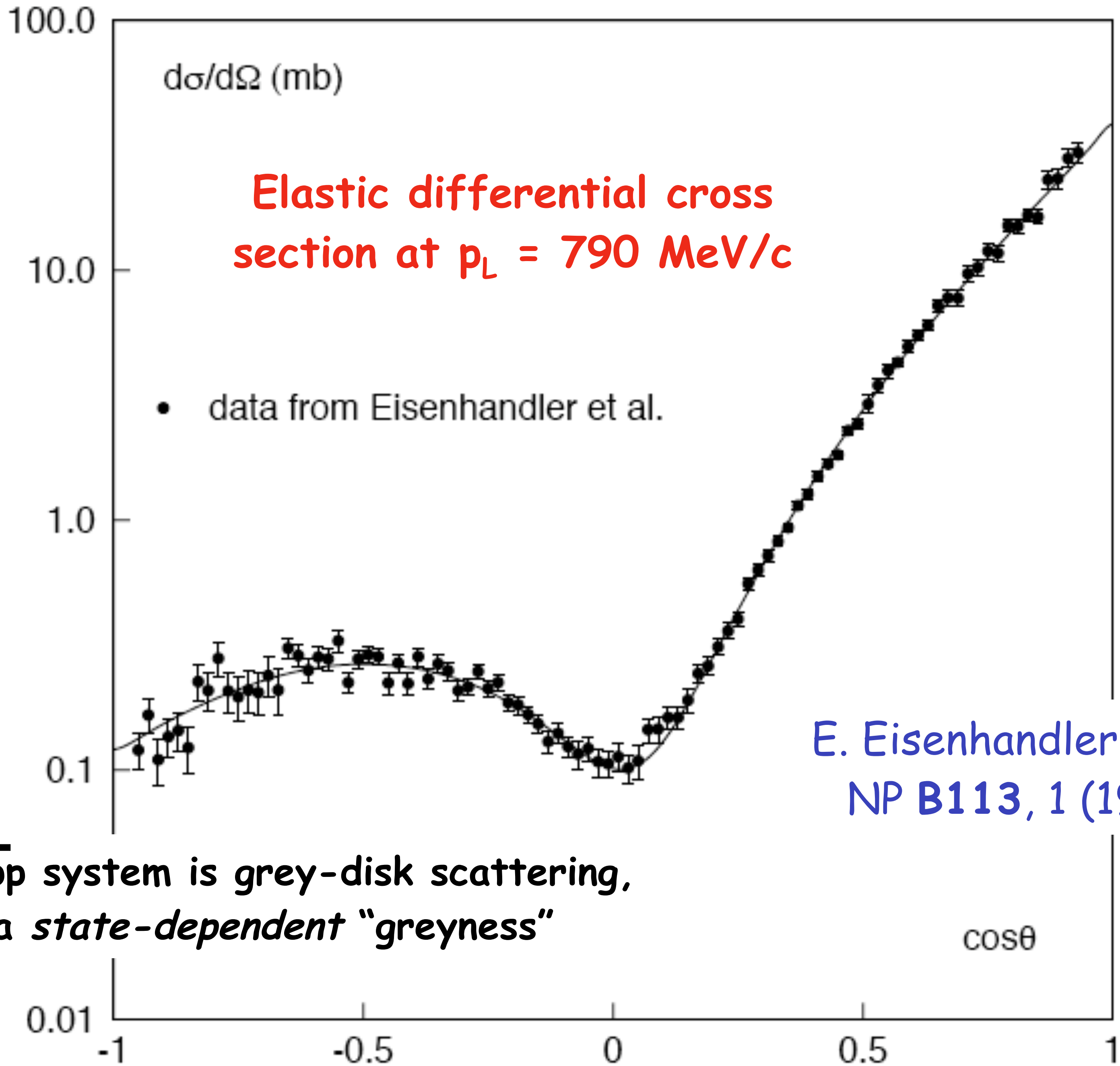
Reasonable quality; 17% rejected

PWA has χ^2/N_{data} = 1.085 with 30 parameters

Only ~25% of the data comes from LEAR

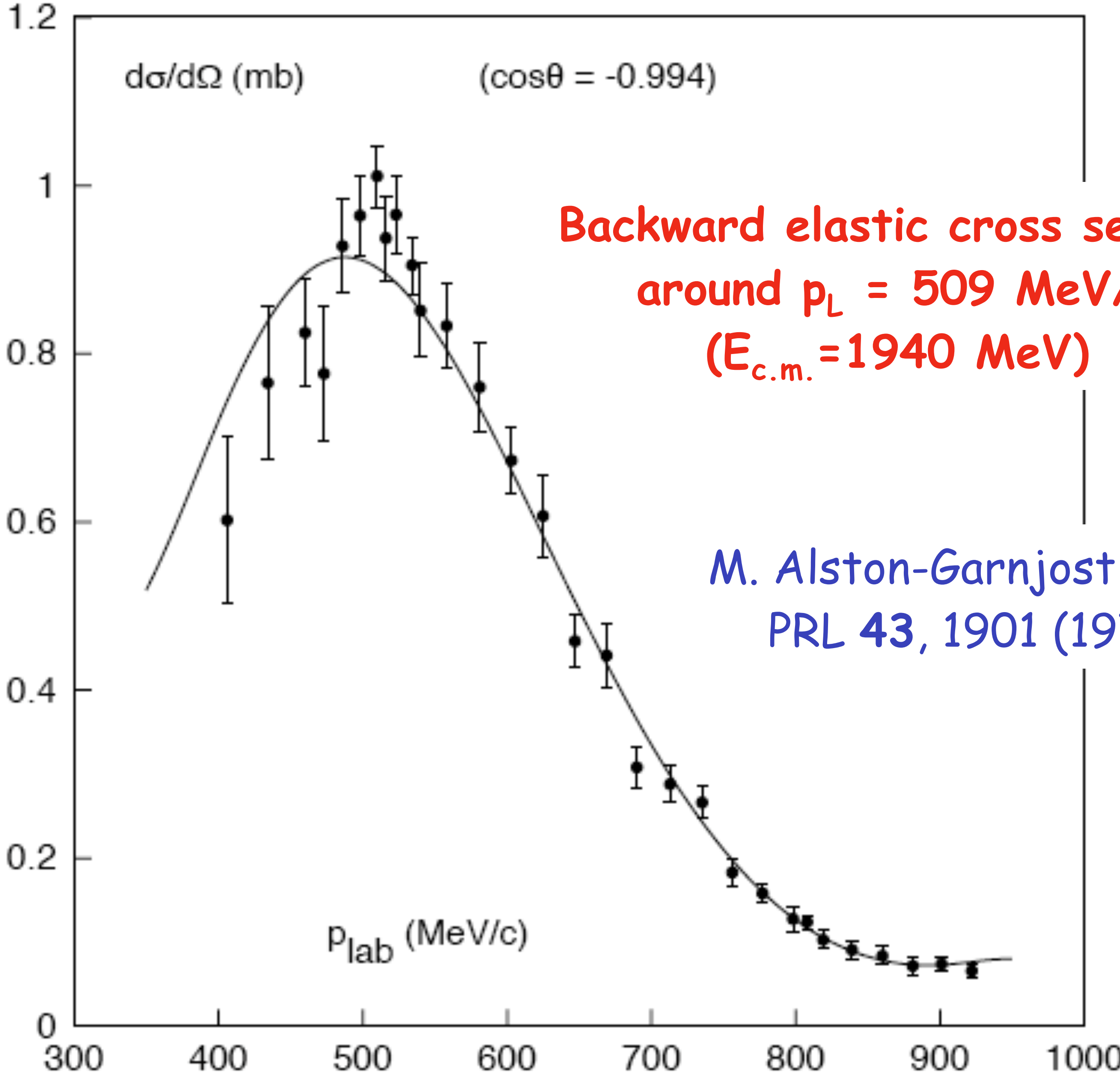
R. Timmermans *et al.*, PRC **50**, 48 (1994).

Elastic differential cross section at p_L = 790 MeV/c



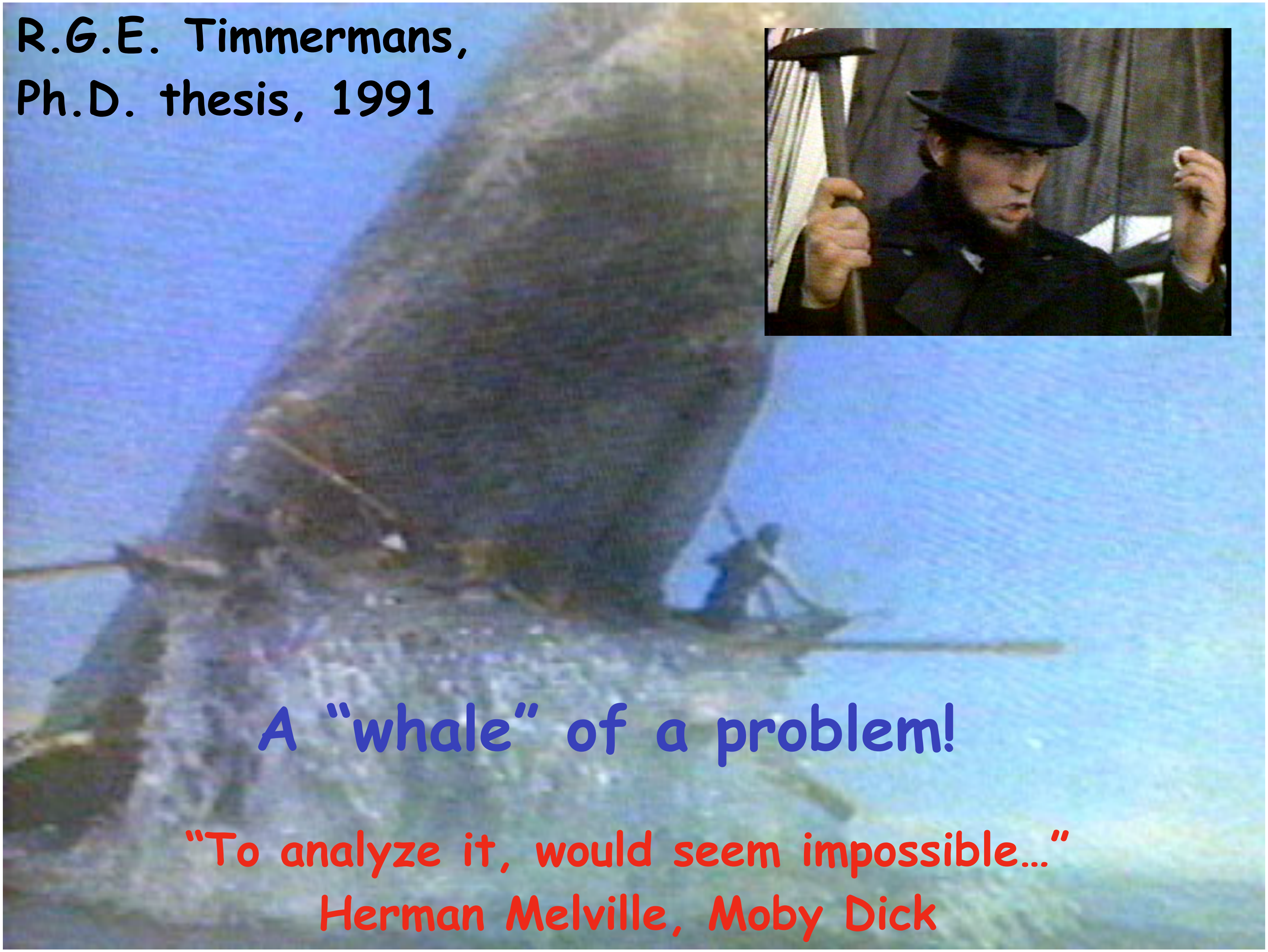
E. Eisenhandler *et al.*,
NP **B113**, 1 (1976)

The $\bar{p}p$ system is grey-disk scattering, with a *state-dependent* "greyness"



Backward elastic cross section around p_L = 509 MeV/c ($E_{c.m.}$=1940 MeV)

M. Alston-Garnjost *et al.*, PRL **43**, 1901 (1979)


R.G.E. Timmermans,
Ph.D. thesis, 1991
A "whale" of a problem!
"To analyze it, would seem impossible…"
Herman Melville, Moby Dick

Amplitude analysis vs. PWA

In a single-energy *Amplitude Analysis* one must determine five complex amplitudes a(θ), b(θ), c(θ), d(θ), e(θ), for every angle at a fixed energy. Therefore one needs to perform, *for each energy*, 9 experiments, at all angles! This was never done…

In a *single-energy* phase-shift analysis ("kid's stuff") one needs for each value of the total angular momentum J:

- **2.5 parameters (on average) for pp,**
- **5 parameters for np (2 for J=0),**
- **20 parameters for $\bar{p}p$ (5 for J=0).**

In order to perform an *energy-dependent* PWA, one needs *much less:* 21 parameters for all pp scattering data below 350 MeV, and *30 parameters for all $\bar{p}p$ data below 925 MeV/c*

Cf. R. Timmermans *et al.*, PRC **52**, 1145 (1995).

"Spin physics"

The darling of every PAC, worldwide…

Proton-proton PWA below T_L = 350 MeV, # data = 1787:

- $d\sigma/d\Omega$, A_y : # = 1381
- "spin data" : # = 406

χ^2_{min}/N_{data} = 1.00

Now do a PWA of only the 1381 "non-spin" data:

- χ^2_{min} = 1404 , or χ^2_{min}/N_{data} = 1.02

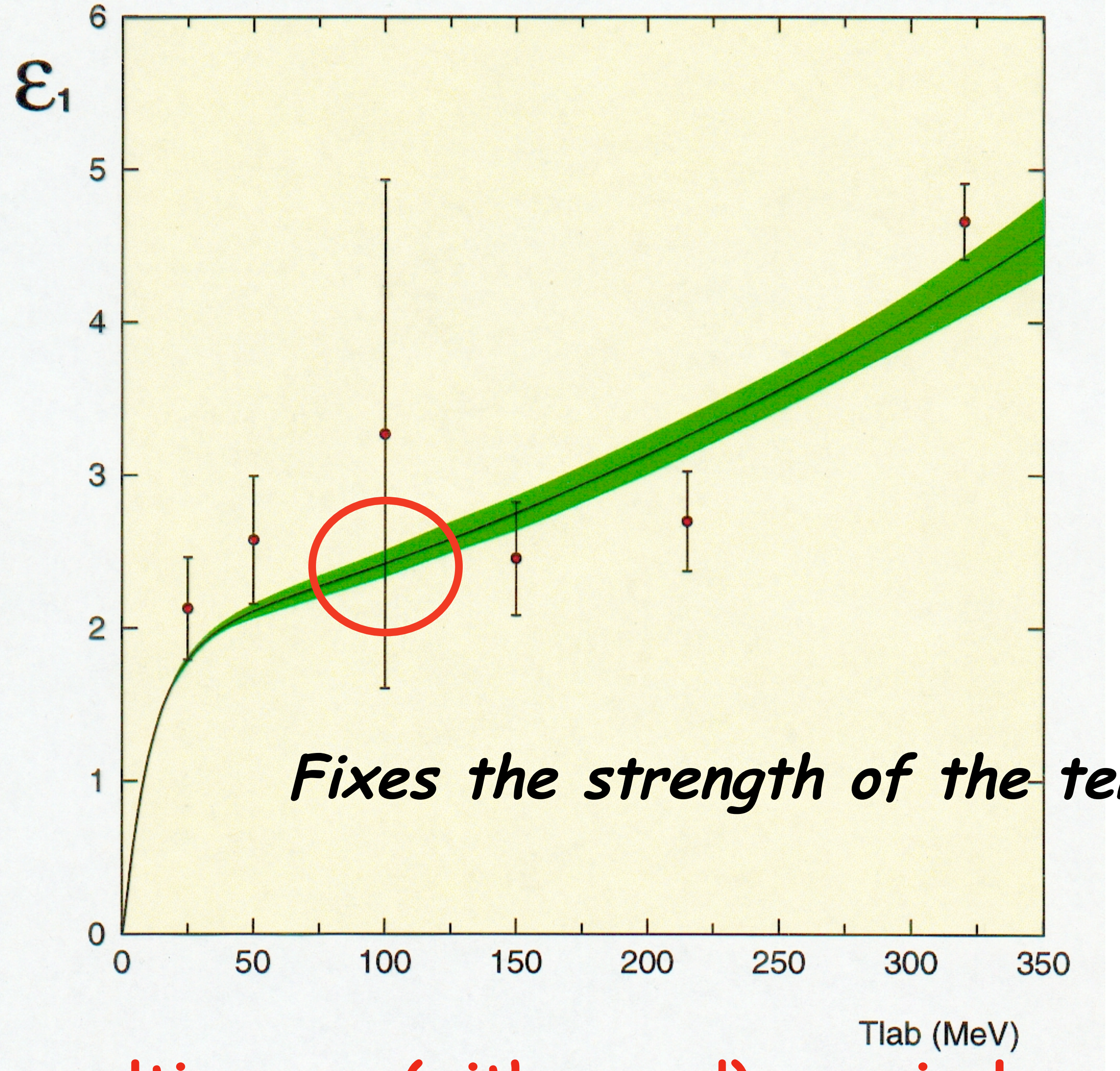
Use this PWA of the "non-spin" data to *predict* the "spin data":

- for N_{data} = 1787 we get χ^2/N_{data} = 1.23 !

How many "spin data" are actually *useful* ?

- 406 "spin data" with χ^2 = 800
- 64 points (8 sets) have χ^2 = 477; 342 points have χ^2 = 321!

The $^3S_1$-$^3D_1$ mixing parameter ε_1 *after* the PWA



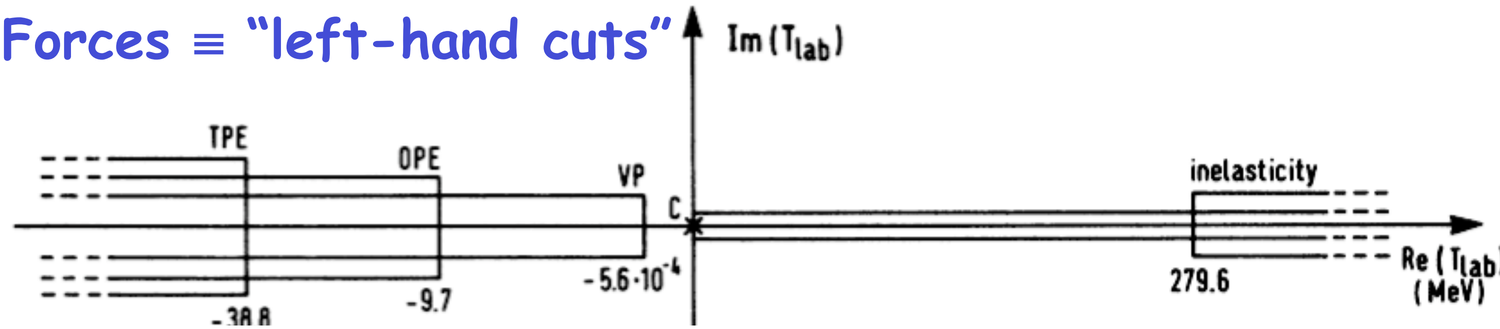
Fixes the strength of the tensor force

Note: multienergy (with error!) vs. single-energy

Analyticity of the S-matrix

- PWA is impossible without theory input
- Need good theory for the *energy dependence* of the amplitudes

Forces ≡ “left-hand cuts”



Rapid energy dependence ← nearby cuts ← *long-range* interaction

Slow energy dependence ← far-away cuts ← *short-range* interaction

Strategy:

(i) Calculate long-range interaction V_L from field theory

(ii) Treat short-range interaction V_S *completely general*

Coulomb	$1/r$
Rel.corr.+2γ	$1/r^2$
Magn.mom.	$1/r^3$
Vac.pol.	$\exp(-2m_e r)/r^{3/2}$
OPE	$\exp(-m_\pi r)/r$
TPE	$\exp(-2m_\pi r)/r^{5/2}$

PWA: Implementation

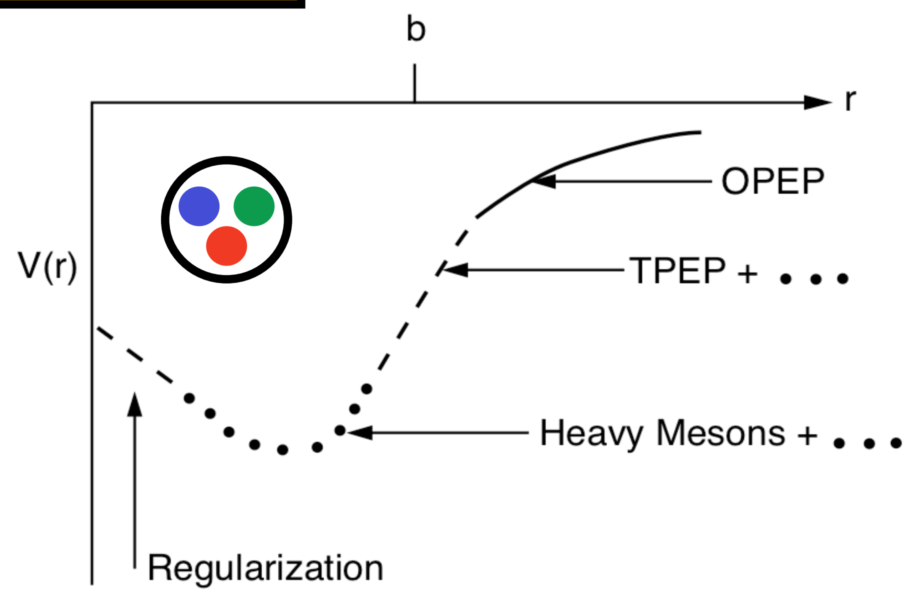
Do not parametrize $\delta_L(E)$ directly

Instead, parametrize a boundary condition* at r=b:

$$BC = b[d\psi/dr]\psi^{-1}|_{r=b}$$

as analytic function of energy:

$$V_S = C_0 + C_2 p^2 + C_4 p^4 + \ldots$$



Explicit long-range interaction:

$$V_L(r) = V_{EM}(r) + V_{OPE}(r) + V_{\chi TPE}(r)$$

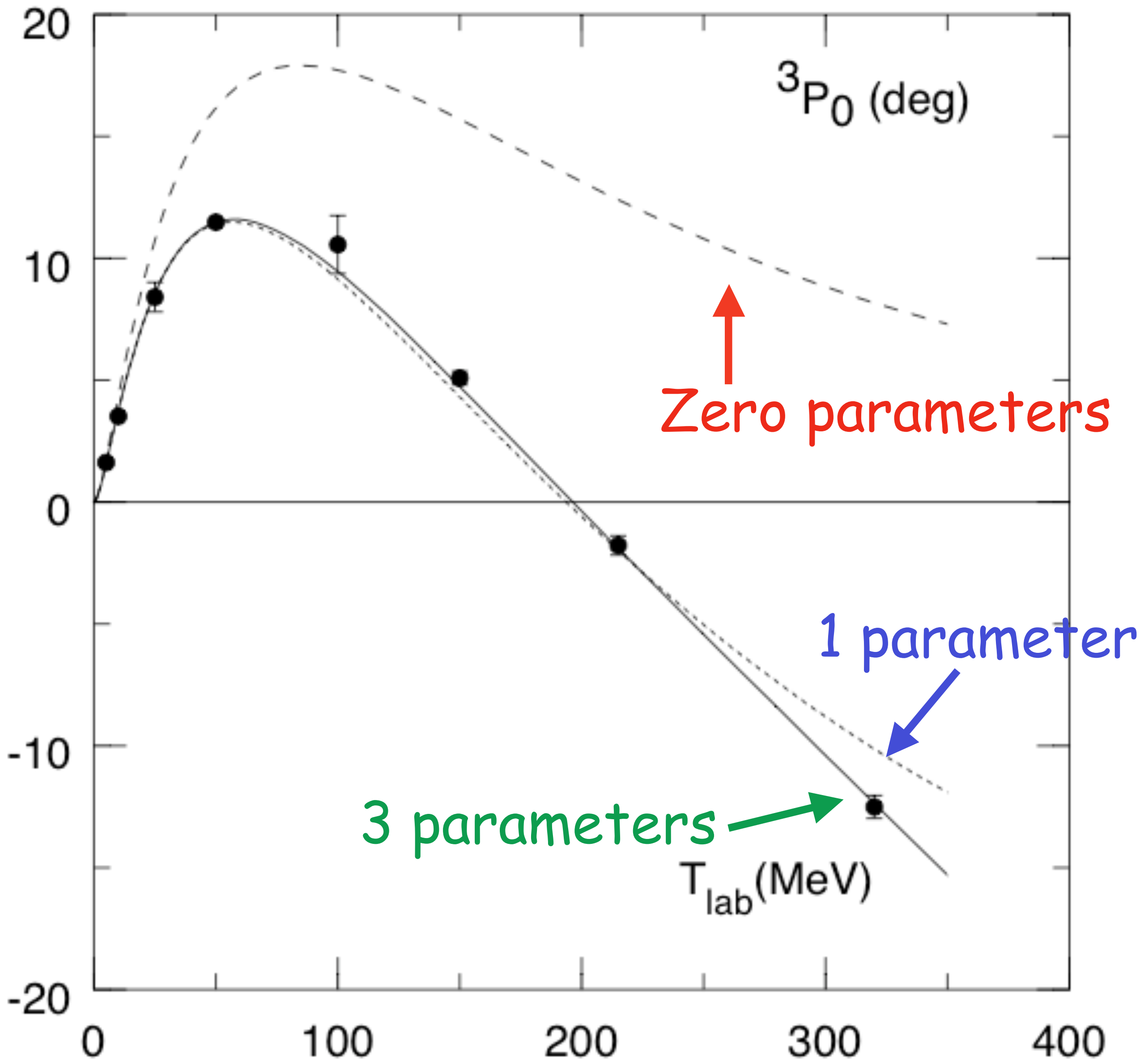
$$V_{EM} = V_C + V_{2\gamma} + V_{VP} + V_{MM}$$

Inelasticity $\eta_L(E)$: complex BC

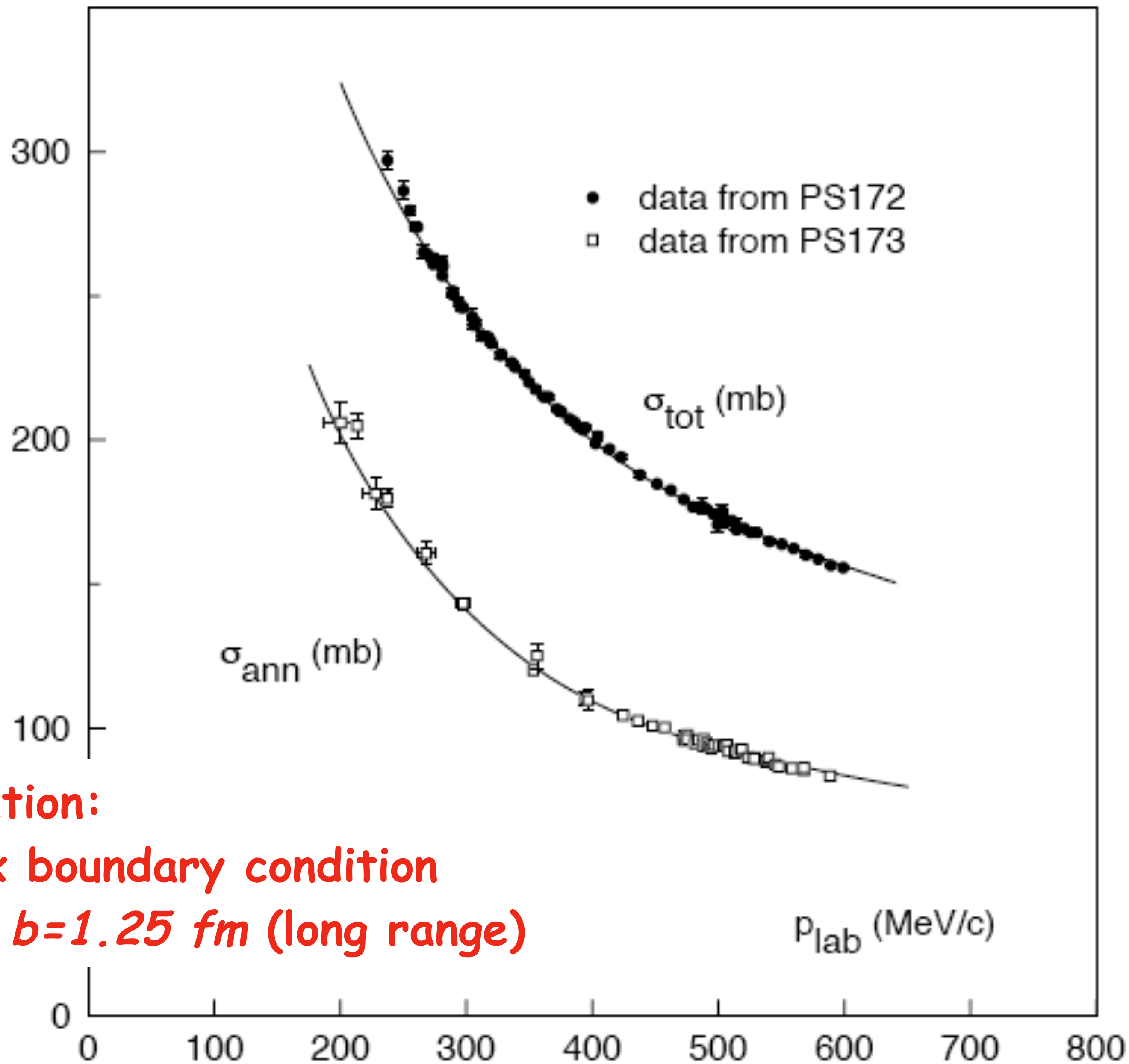
<u># BC parameters *e.g.* pp PWA:</u>

$^1S_0$: 4			$^3P_0$: 3	$^3P_1$: 2
$^1D_2$: 2	$^3P_2$: 3	ε_2: 2	$^3F_2$: 1	$^3F_3$: 3
$^1G_4$: 1	$^3F_4$: 2	ε_4: -	$^3H_4$: -	$^3H_5$: -
$^1I_6$: -	etc.		Total: #=21	

* *Cf.* R.L. Jaffe & F.E. Low, PRD **19**, 2105 (1979).



The details of the short-range interaction do not matter (cf. EFT)



Annihilation:

Complex boundary condition

Radius: *b=1.25 fm* (long range)

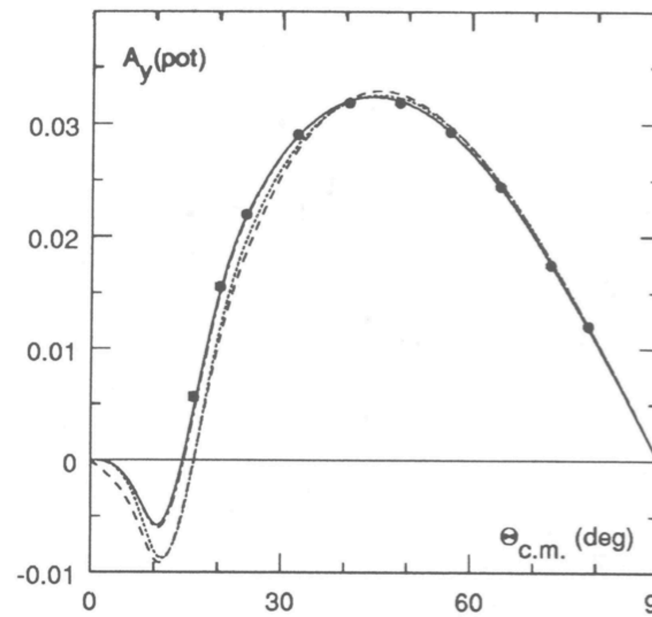
Long-range EM effects

Magnetic-moment interaction:

$V_{MM} = -\alpha[\mu_p^2 S_{12} + (6+8\kappa_p) L \cdot S]/4m^2 r^3$

- Charge & magn. mom. → spin-orbit
- Magn. mom. & magn. mom. → tensor

- In pp: $\Delta\chi^2_{min} \approx +400$, so *20 s.d.*

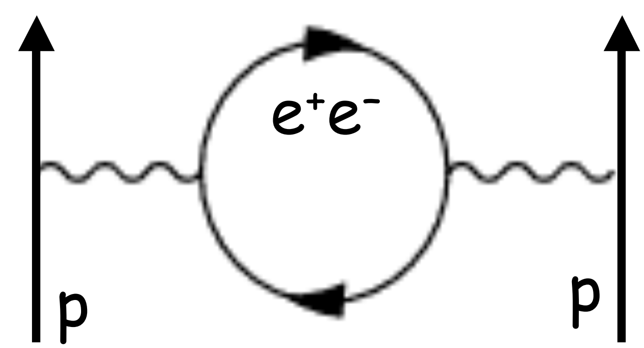


Vacuum polarization:

$V_{VP} = \alpha' I(r)/r \approx \alpha\alpha' \exp(-2m_e r)/r^{3/2}$

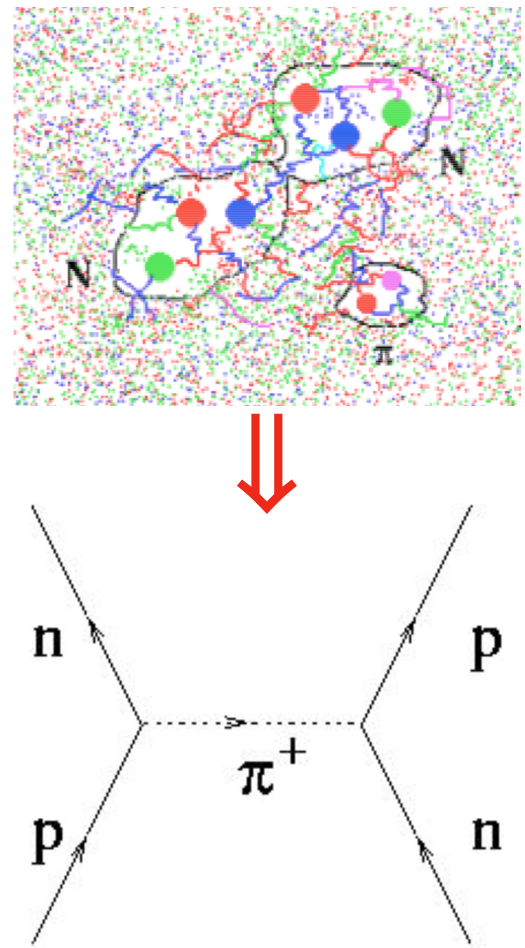
- Enhances Coulomb force V_C
- Long-range: $1/2m_e \approx 200$ fm
- Relevant in proton-proton $^1S_0$ wave

- In pp: $\Delta\chi^2_{min} \approx +215$, so *15 s.d.*



One-pion exchange: the "glue" of nuclei

Strong interaction: $V_{nuc}(r) = V_{OPE} + V_{\chi TPE} + V_{\gamma\pi}$



One-pion exchange ($x = m_\pi r$):

$$V_{OPE} = f^2 m_\pi [\sigma_1 \cdot \sigma_2 + \xi(x) S_{12}] \exp(-x)/3x$$

- Charge-dependent:
  - $m(\pi^0)$ vs. $m(\pi^\pm)$
  - $f(pp\pi^0)$ vs. $f(nn\pi^0)$ vs. $f(np\pi^\pm)$

Best value: $f^2 = 0.0750(9)$*

Goldberger-Treiman relation: $f_{NN\pi}/m_\pi = g_A/F_\pi$

i.e. the "discrepancy" is only 1-2%

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* Or $g^2/4\pi \approx 13.6$

"Seeing" one-pion exchange

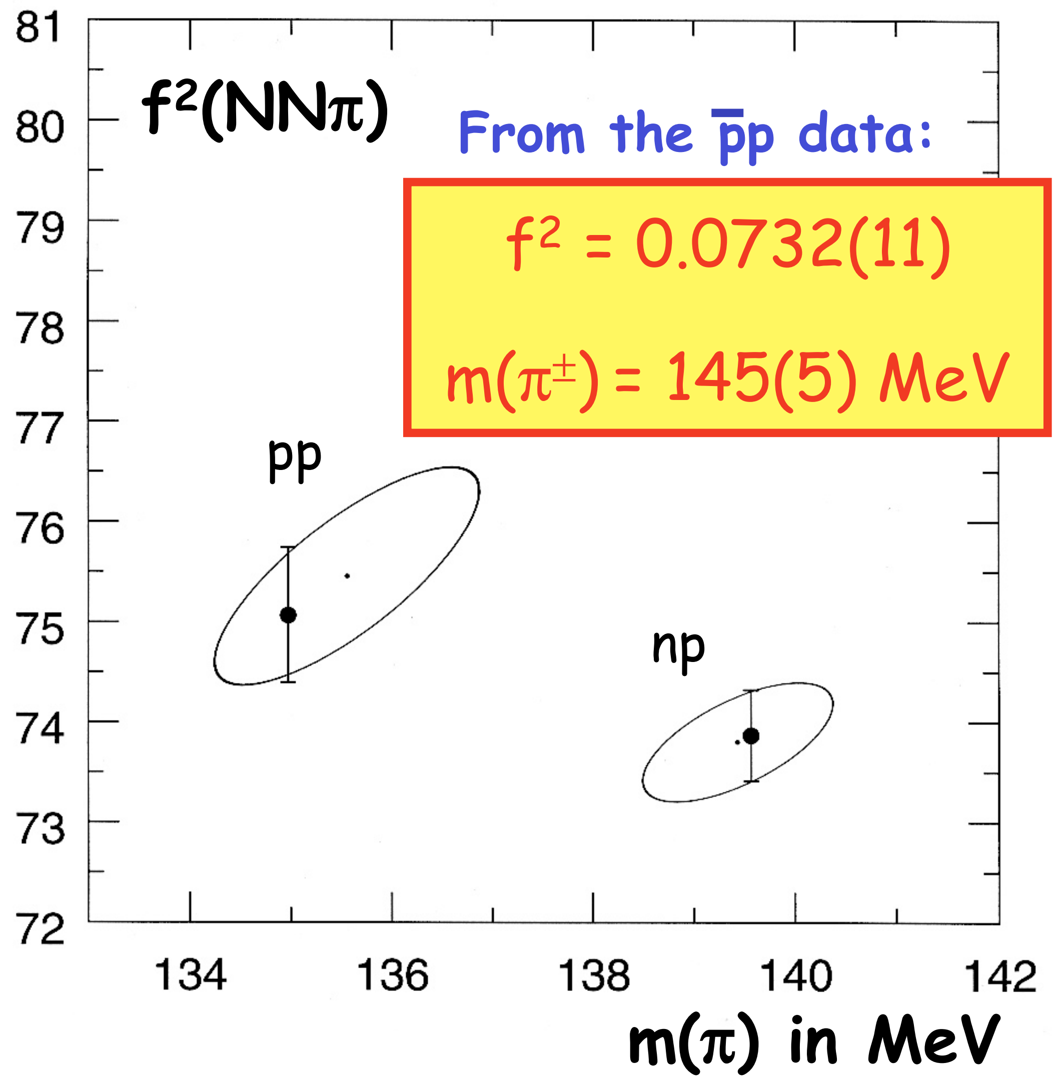
The coupling constant $f^2(NN\pi)$ is determined *at the pion pole* from the long-range OPE

We can also *fit* the pion masses from the pp and the np data:

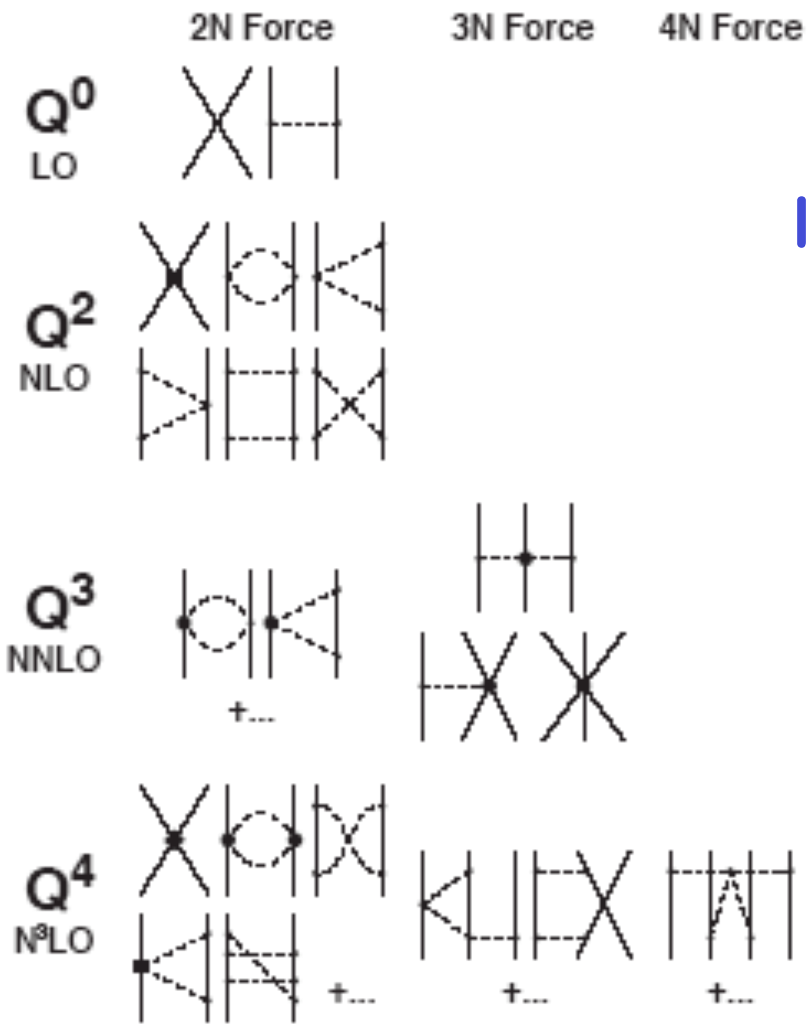
$m(\pi^0) = 135.6(10)$ MeV

$m(\pi^\pm) = 139.6(13)$ MeV

No significant evidence, yet, for isospin violation:
$f(pp\pi^0) \approx f(nn\pi^0) \approx f(np\pi^\pm)$



How the connect nuclear physics to QCD: A pipedream?



Lattice QCD ⇒

low-energy constants of χPT

⇓

NN & few-nucleon systems

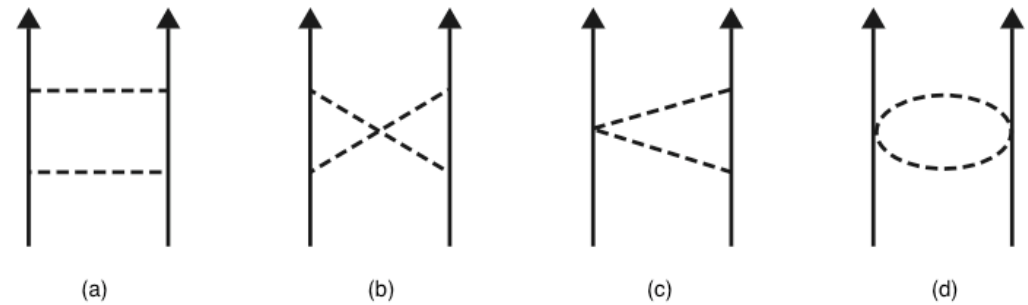
⇒ χPT: *controlled* expansion

⇓

Nuclear structure ⇒

via many-body EFT?

"Seeing" chiral two-pion exchange



With long-range OPE+χTPE, an excellent χ^2/data ~1 is possible! (no need for the "sigma-meson")

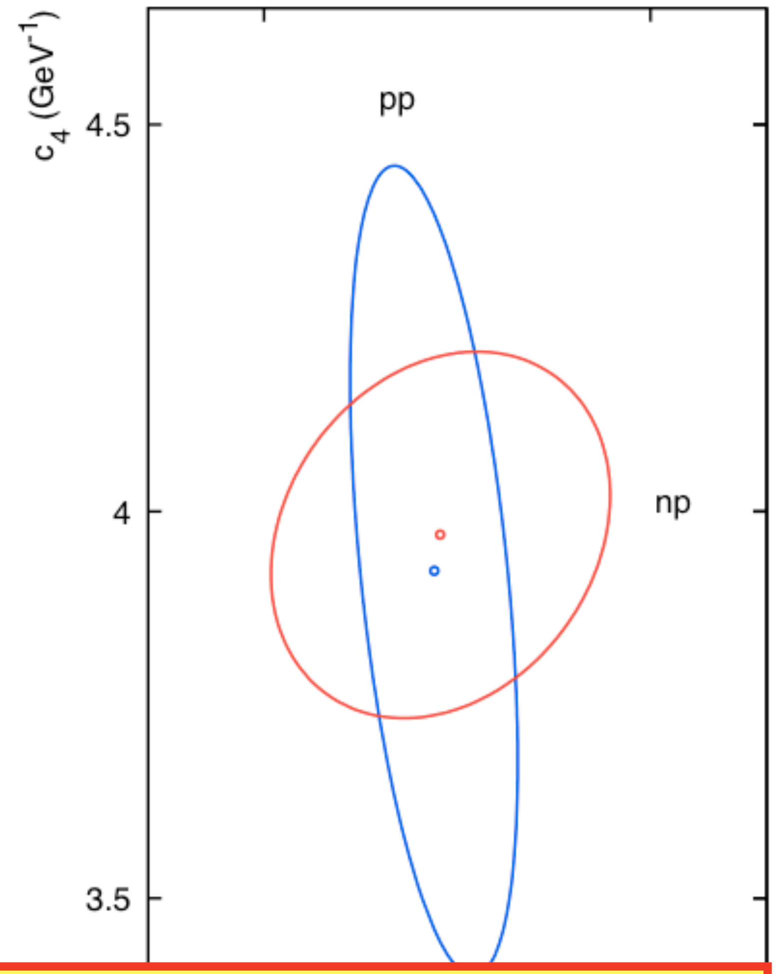
NN$\pi\pi$ coupling constants:

c_3 = -4.78(10)/GeV

c_4 = +3.96(22)/GeV

Pion mass from long-range TPE:

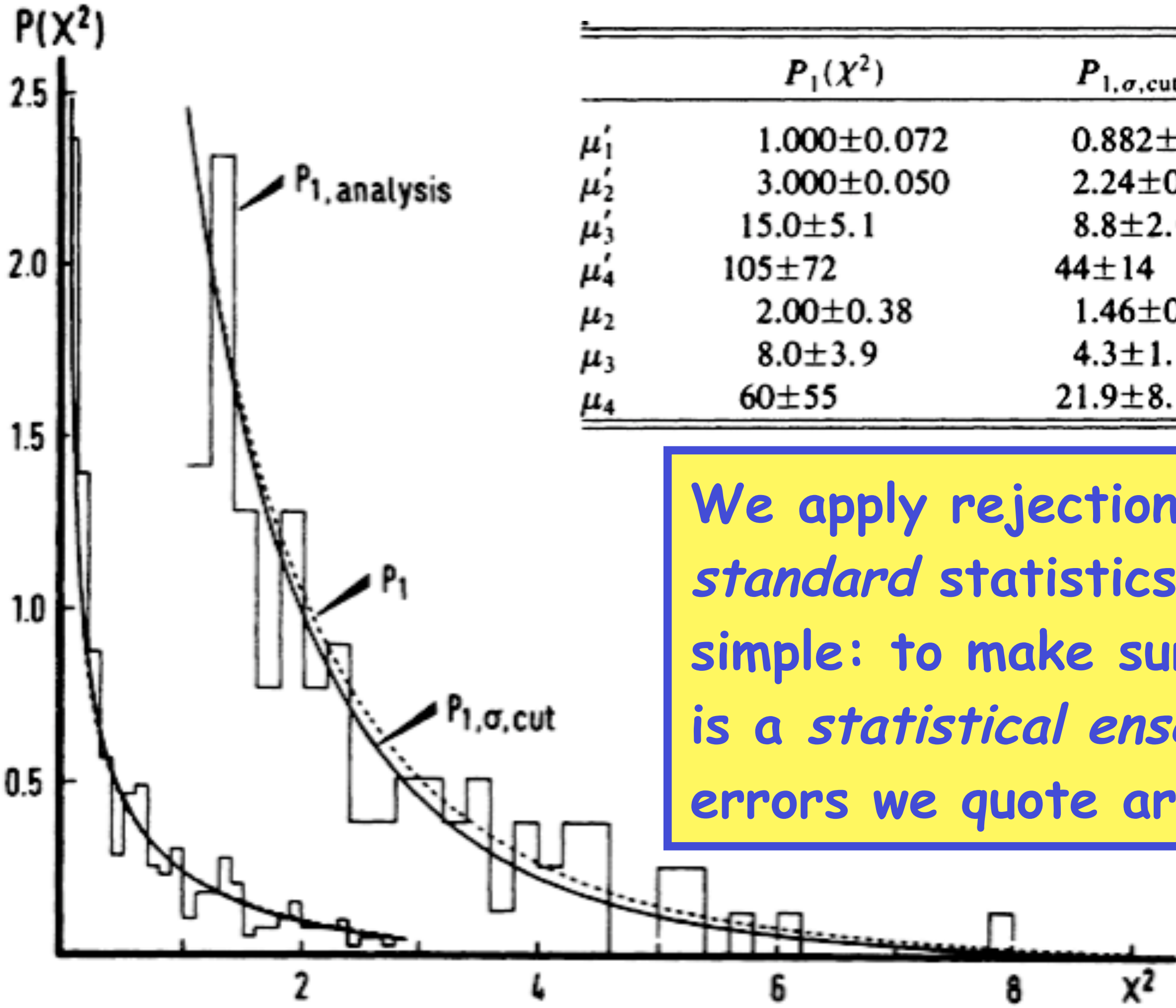
m_π = 128(9) MeV



OPE + (charge-conj.) TPE provides an excellent long-range NN interaction.

M.C.M. Rentmeester *et al.* (1999-2005).

Lies, damned lies & statistics!

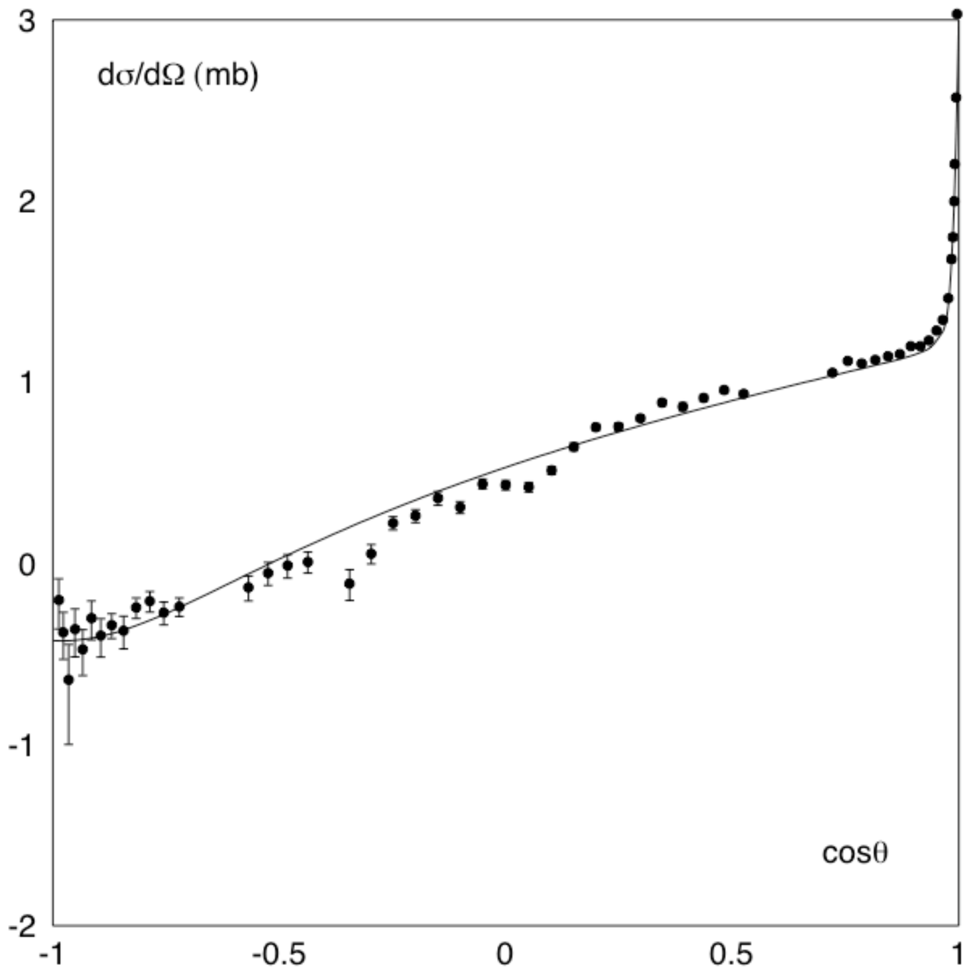


	$P_1(\chi^2)$	$P_{1,\sigma,\text{cut}}(\chi^2)$	$P_{1,\text{analysis}}(\chi^2)$
μ_1'	1.000±0.072	0.882±0.061	0.883
μ_2'	3.000±0.050	2.24±0.32	2.24
μ_3'	15.0±5.1	8.8±2.0	8.5
μ_4'	105±72	44±14	40
μ_2	2.00±0.38	1.46±0.23	1.46
μ_3	8.0±3.9	4.3±1.3	3.9
μ_4	60±55	21.9±8.7	18.3

We apply rejection criteria based on *standard* statistics. The reason is very simple: to make sure that the database is a *statistical ensemble* and thus that errors we quote are really statistical!

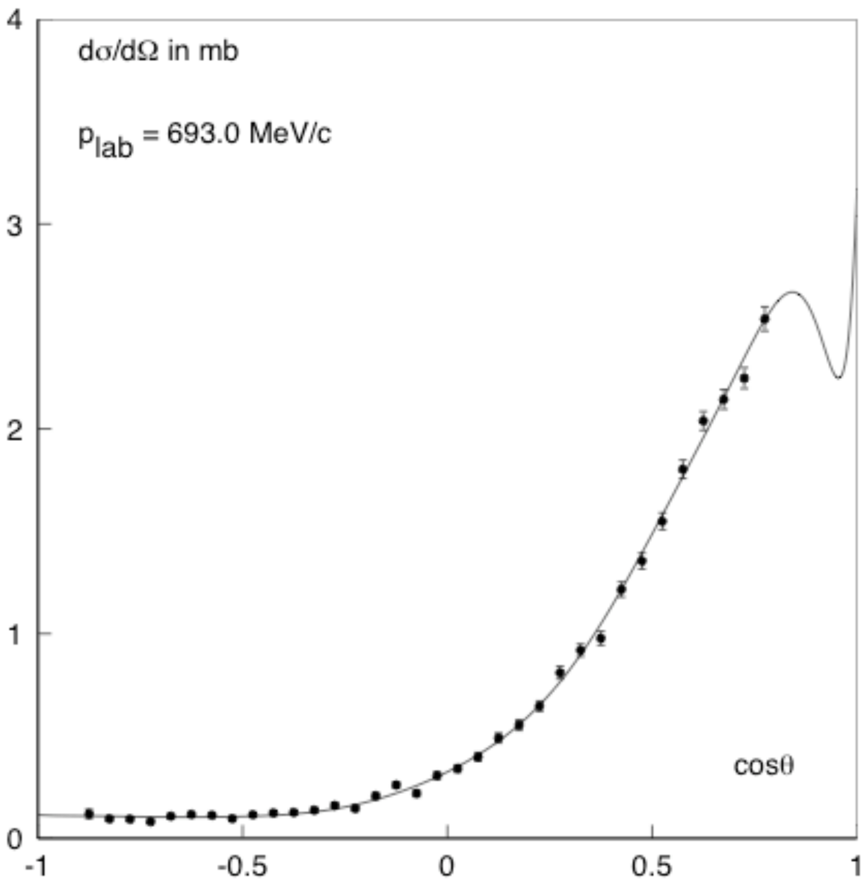
We do not determine if expt's are right or wrong, but we do decide whether they are statistically acceptable, yes or no

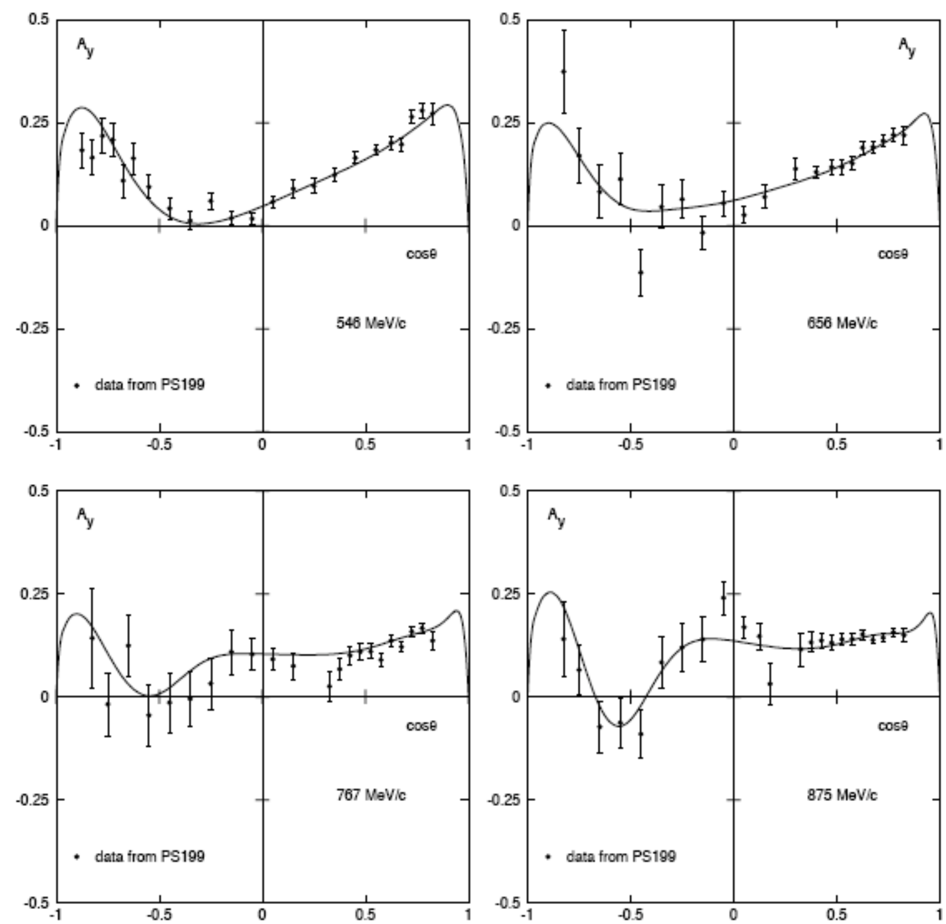
Problematic data sets



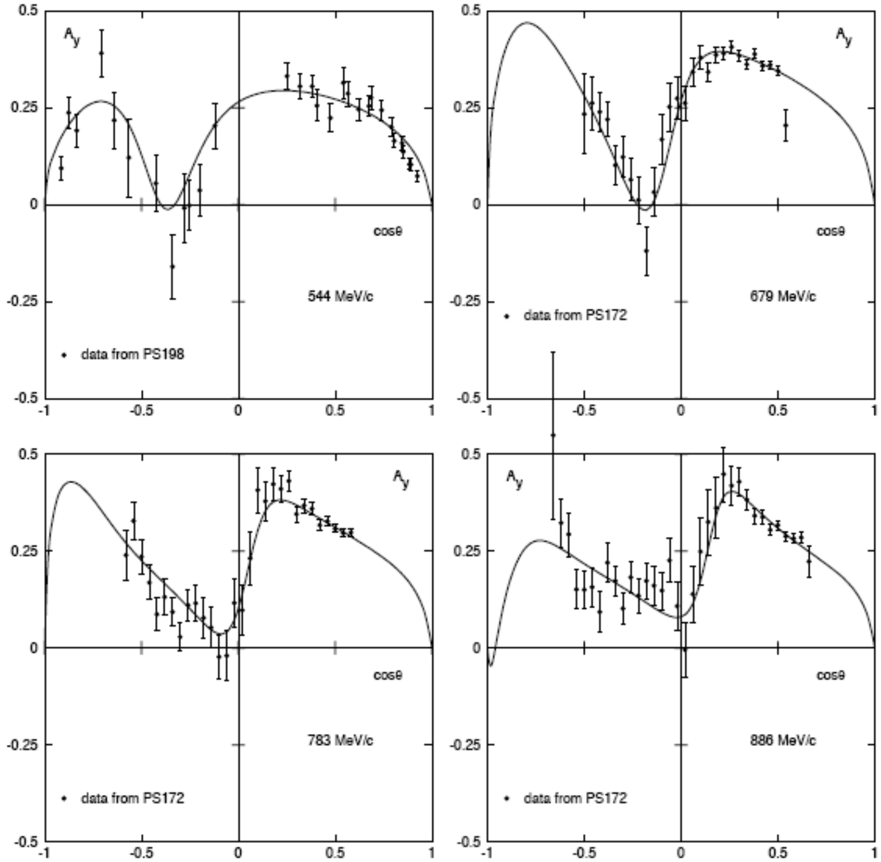
Group	χ^2	# data
PS172	504	84
PS173	505	173
PS198	1743	84
KEK	3096	173

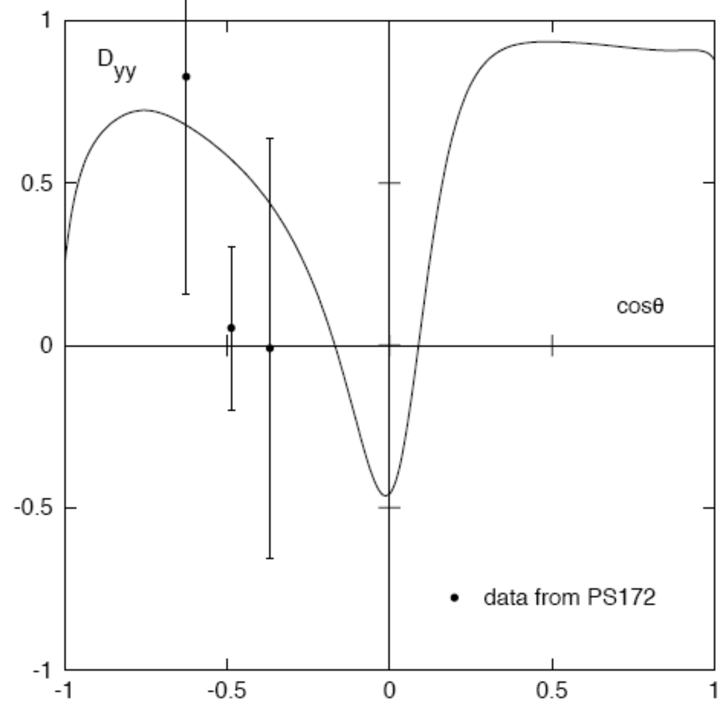
The charge-exchange reaction $\bar{p}+p\rightarrow \bar{n}+n$





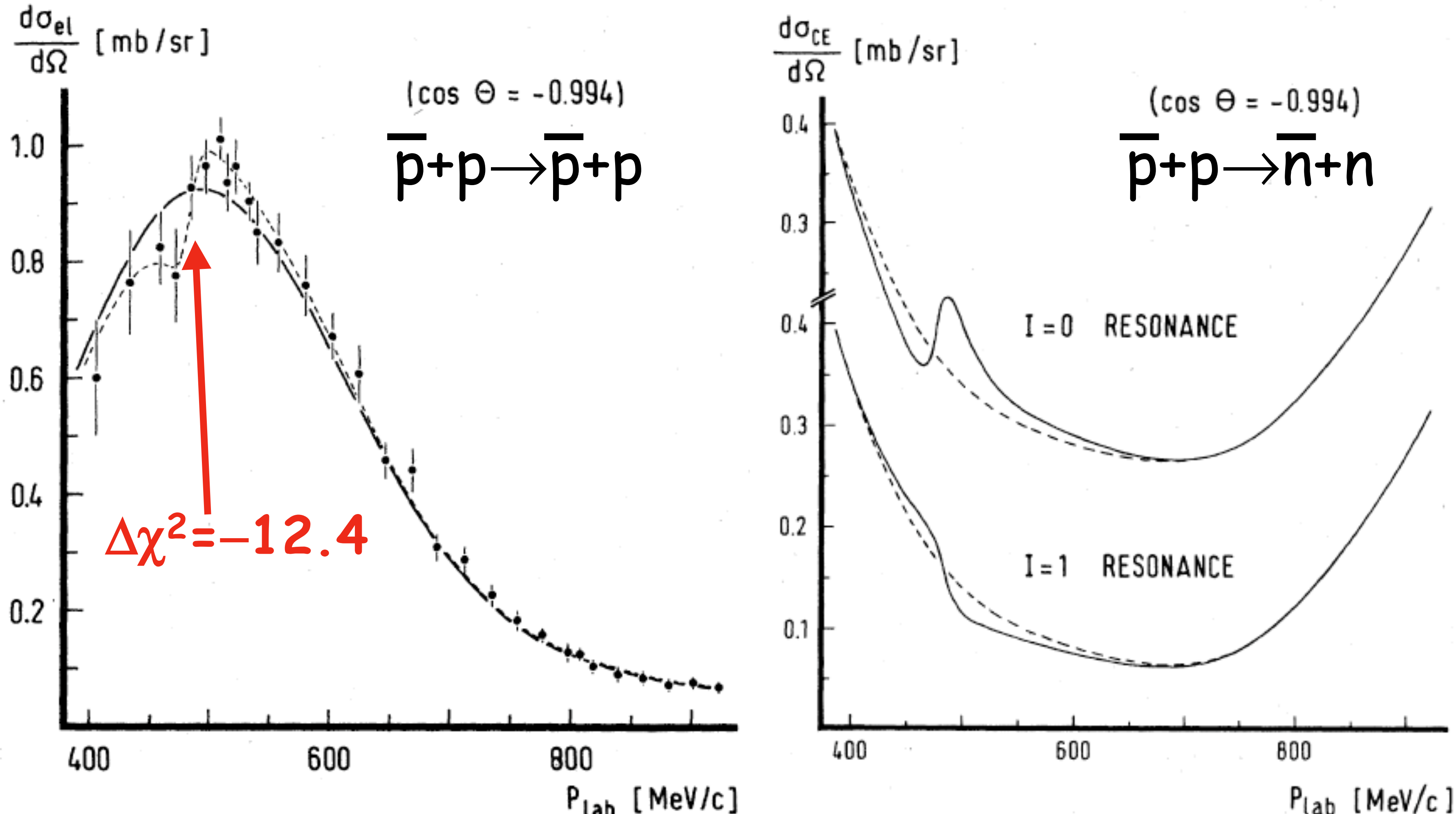
Spin effects in elastic scattering





p_L (MeV/c)	σ_{tot} (mb)	$\Delta\sigma_T$(mb)	$\Delta\sigma_L$(mb)	$\Delta\sigma_T/\sigma_{tot}$ (%)	$\Delta\sigma_L/\sigma_{tot}$ (%)
200	314.8	-91.0	-19.4	-28.9	-6.2
400	194.0	-45.6	-51.8	-23.5	-26.7
600	151.8	-31.5	-58.6	-20.8	-38.6
800	128.5	-25.8	-54.1	-20.1	-42.1

PWA: Hunting resonances



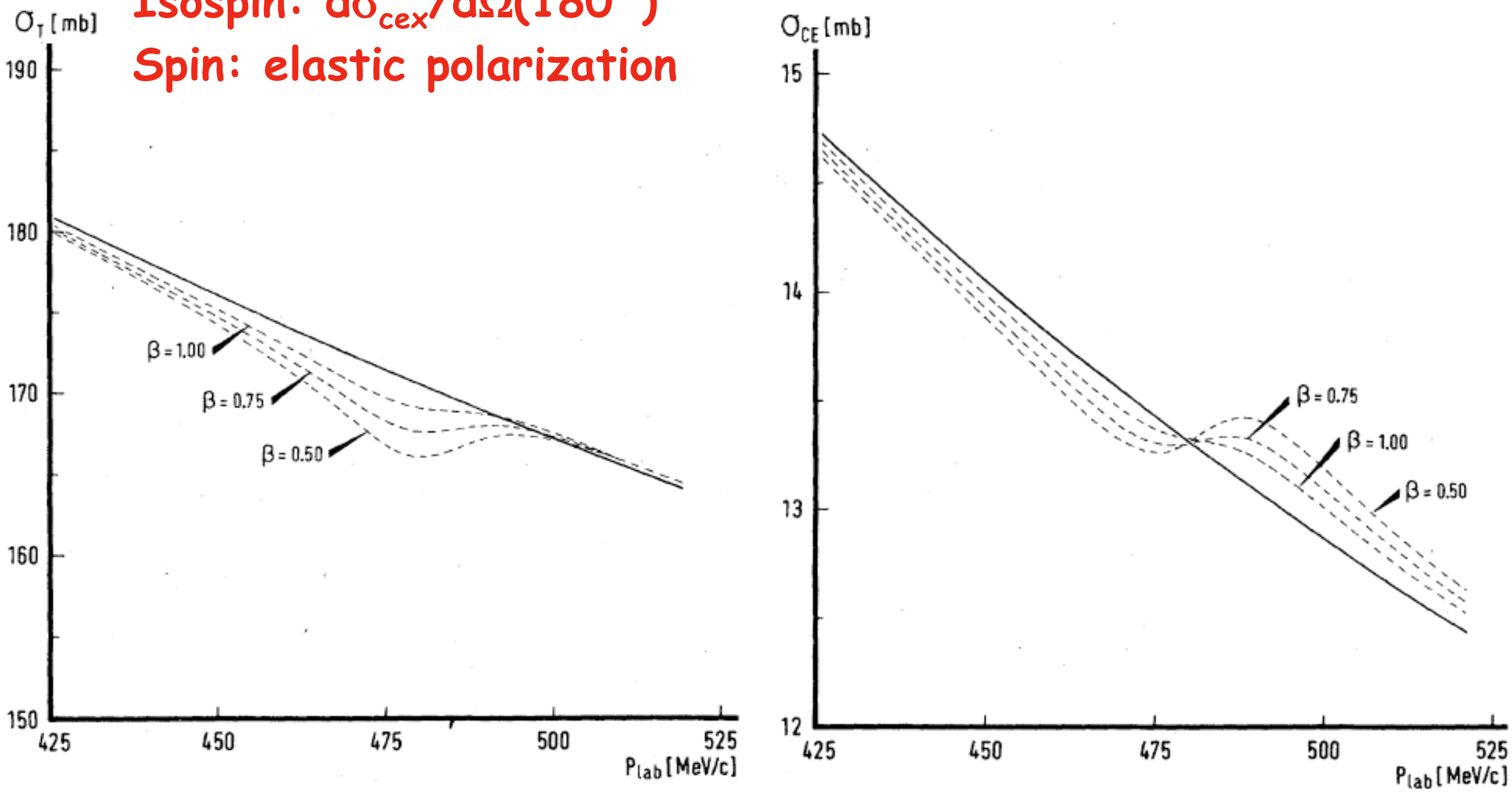
Including a resonance in the $^{11}D_2$ partial wave, with E_R=1934(3) MeV, Γ=6(4) MeV

P. Timmers *et al.*, PRD **31**, 99 (1985).

Parity: σ_{tot} and $d\sigma_{el}/d\Omega(180°)$

Isospin: $d\sigma_{cex}/d\Omega(180°)$

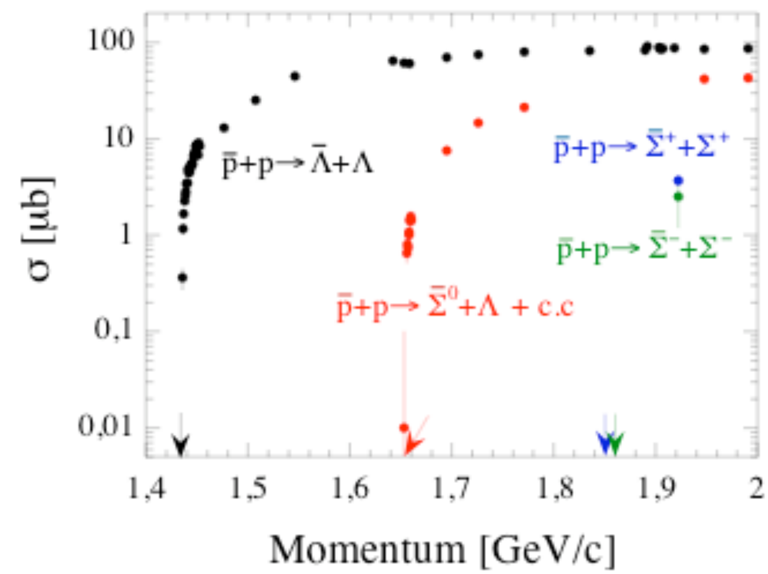
Spin: elastic polarization



Caveat: A structure in the backward cross section is not necessarily accompanied by a pronounced effect in the total cross section!

PS185@LEAR

	p_L MeV/c	ΔQ	ΔS
$\bar{p}p$	0	0	0
$\bar{n}n$	100	1	0
$\bar{\Lambda}\Lambda$	1435	1	1
$\bar{\Lambda}\Sigma^0$, $\bar{\Sigma}^0\Lambda$	1653	1	1
$\bar{\Sigma}^+\Sigma^+$	1853	0	1
$\bar{\Sigma}^0\Sigma^0$	1871	1	1
$\bar{\Sigma}^-\Sigma^-$	1899	2	1
$\bar{\Xi}^0\Xi^0$	2582	1	2
$\bar{\Xi}^-\Xi^-$	2620	2	2

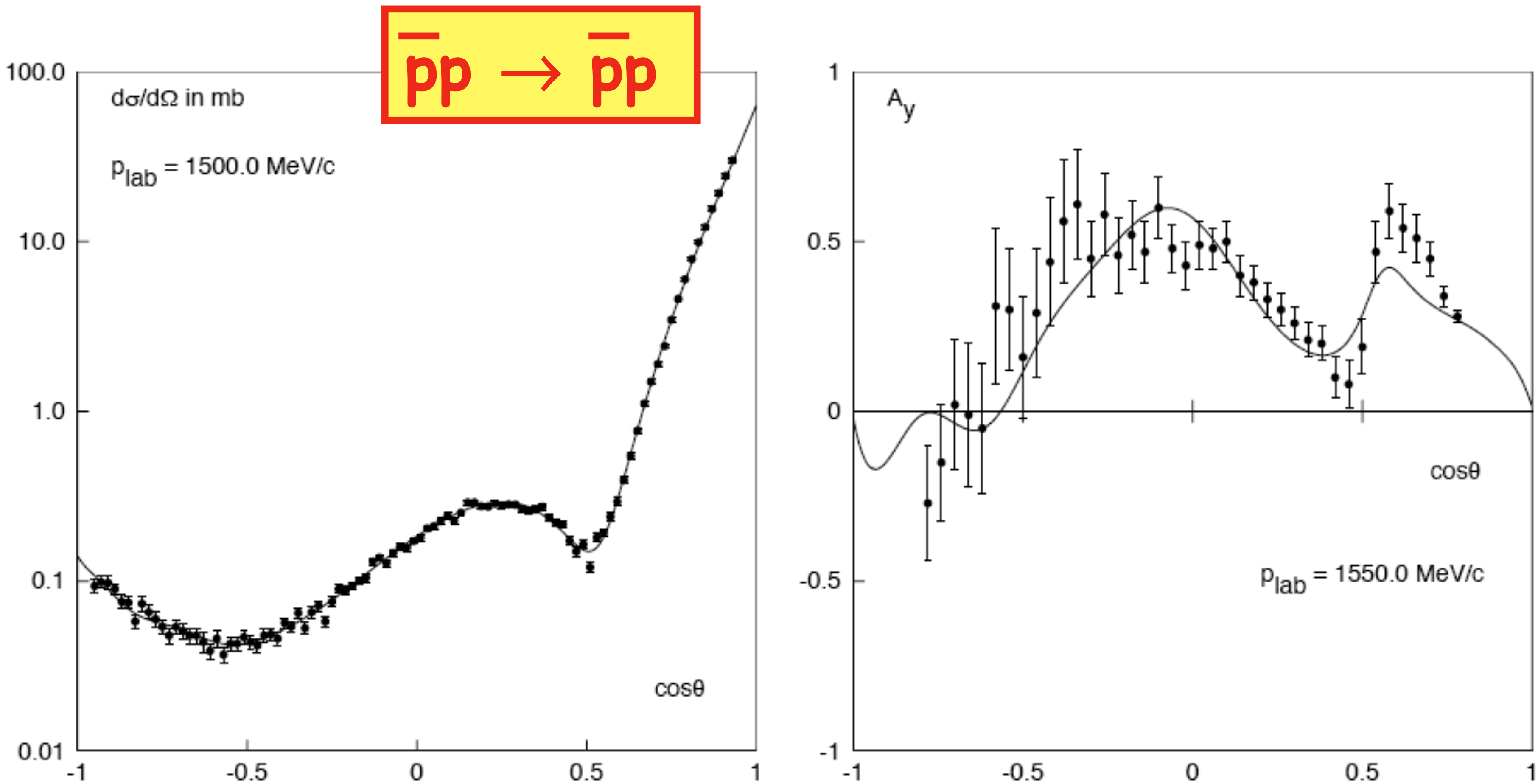


Coupled-channels PWA:

Long-range interaction: Charge-conjugated version of the Nijmegen YN model

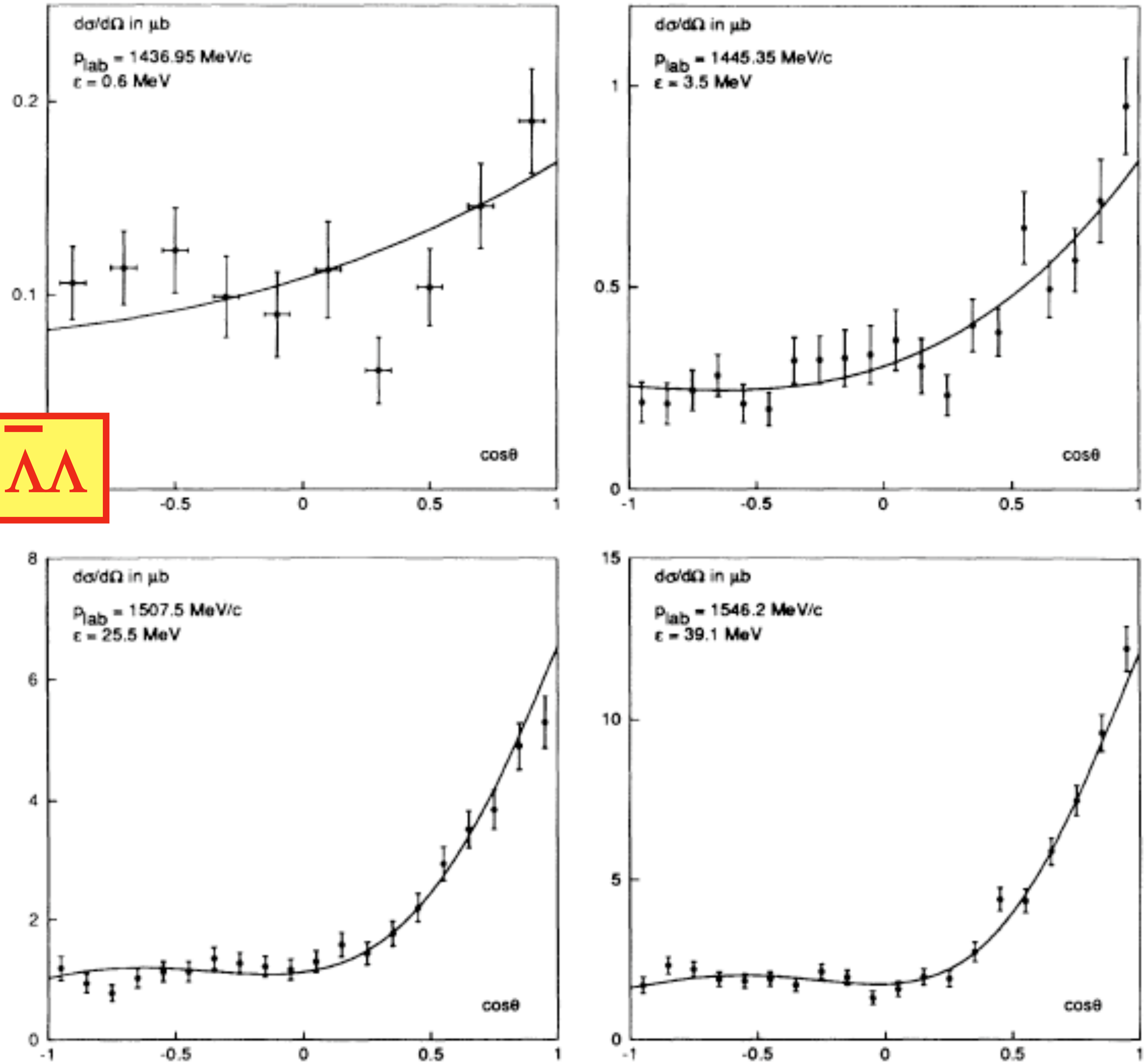
Short-range interaction: Complex boundary condition

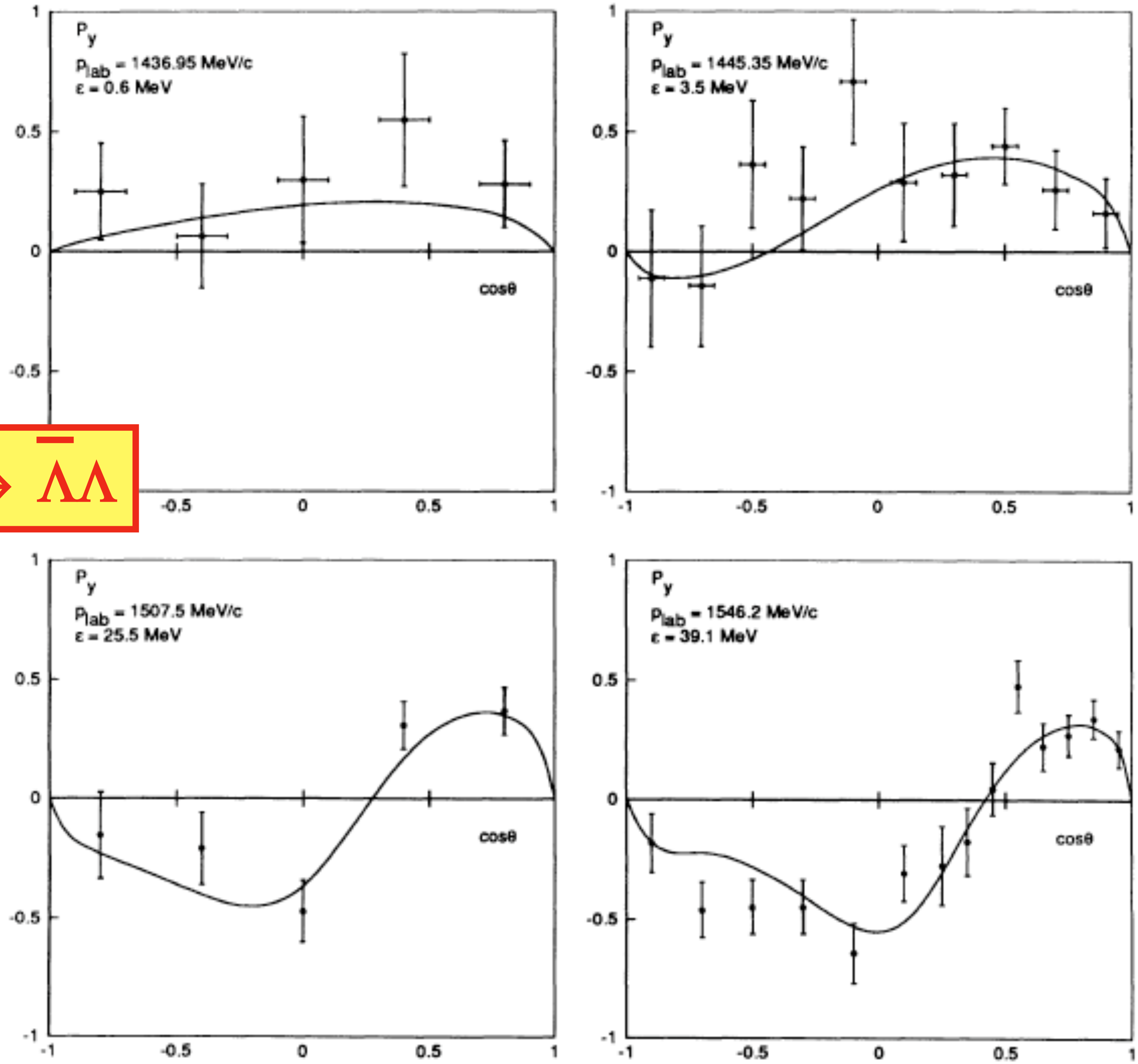
R. Timmermans *et al.*, PRD **45**, 2288 (1992).



Good description of the initial-state interaction

$\bar{p}p \to \bar{\Lambda}\Lambda$





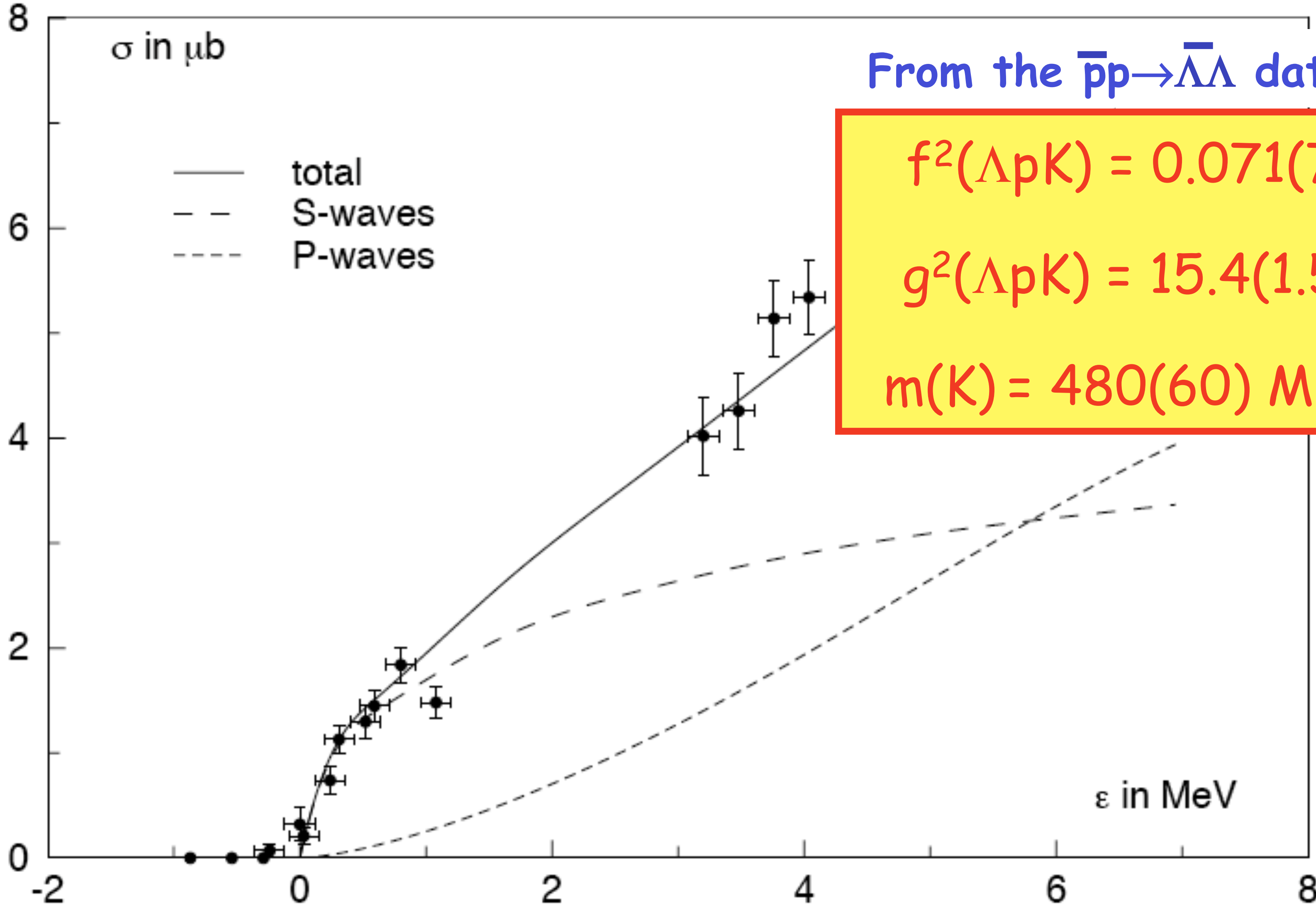
$\bar{p}p \rightarrow \bar{\Lambda}\Lambda$

p_{lab} (MeV/c)	1435.95	1436.95	1445.35	1476.5	1507.5	1546.2
ε (MeV)	0.24	0.59	3.5	14.5	25.5	39.1
$^3D_1 \rightarrow {^3S_1}$	0.89	1.36	2.9	4.2	4.3	4.0
$^3F_2 \rightarrow {^3P_2}$	0.01	0.05	0.7	4.0	6.7	8.9
$^3G_3 \rightarrow {^3D_3}$				1.2	4.0	9.6
$^1S_0$		0.01				
$^1P_1$					0.1	0.1
$^3S_1$	0.08	0.12	0.3	0.5	0.6	0.7
$^3P_0$		0.01	0.1	0.5	0.6	0.7
$^3P_1$	0.01	0.04	0.5	2.9	4.5	5.3
$^3P_2$	0.01	0.03	0.4	2.1	3.7	5.1
$^3D_1$				0.1	0.2	0.5
$^3D_2$				0.2	0.6	1.4
$^3D_3$				0.4	1.3	3.2
$^3F_3$					0.1	0.2
$^3S_1 \rightarrow {^3D_1}$				0.1	0.3	0.8
$^3P_2 \rightarrow {^3F_2}$						0.1
$J \geq 4$					0.2	0.9
Singlet $s=0$	0.00	0.01	0.0	0.1	0.1	0.1
Triplet $s=1$	1.00	1.60	4.9	16.1	27.3	41.3
Total	1.00	1.61	4.9	16.2	27.3	41.4
Experimental	0.84(20)	1.44(32)	4.86(42)	13.8(5)	26.6(7)	44.6(1.5)

$\bar{p}p \rightarrow \bar{\Lambda}\Lambda$

Strong coherent tensor force due to K- and K*-exchange

"Seeing" one-kaon exchange



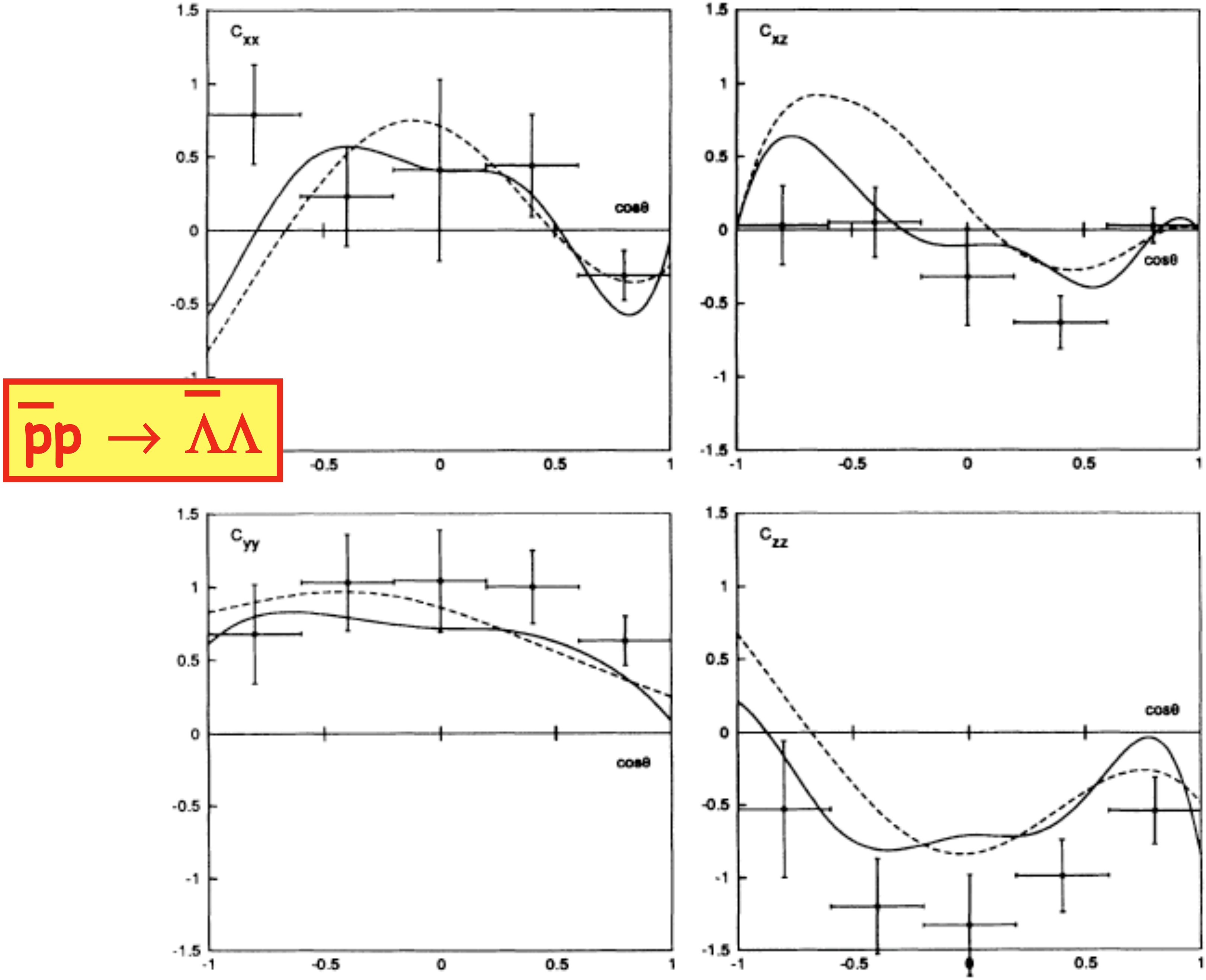
From the $\bar{p}p \to \bar{\Lambda}\Lambda$ data:

$f^2(\Lambda pK) = 0.071(7)$

$g^2(\Lambda pK) = 15.4(1.5)$

$m(K) = 480(60)$ MeV

R. Timmermans *et al.*, PLB **257**, 227 (1991), NP **A585**, 143 (1995).


C_xx
C_xz
C_yy
C_zz
cosθ
p̄p → Λ̄Λ

PWA is an important bridge between expt. and theory

- A high-quality database should be available
- Model-independent theory is needed as input
- The rules of statistics should apply:
  - Input = "raw" database
  - Output = pruned database

Unfortunately, ~15-20% of the data have to be rejected

A coupled-channels antiproton-proton PWA below 2 GeV/c exists & could be extended to HESR energies:

- Provides a very good initial interaction
- Resonance-background separation
- Search for glueballs/hybrids
- Antihyperon-hyperon production