ECT* Trento, 7 July 2006

# QUARKONIA: Notes on **Spectroscopy** and **String Breaking**

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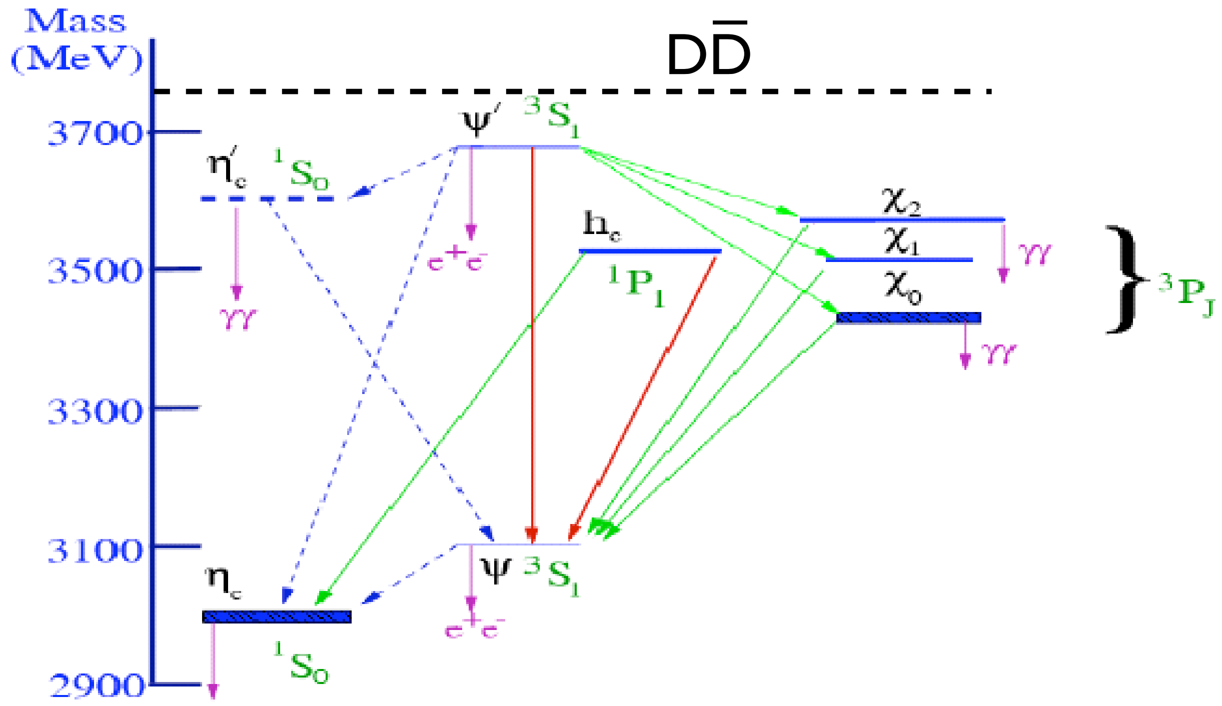
(with: Johannes Eiglsperger, Norbert Kaiser)

- **Potentials**: Confinement plus Gluon Exchange and beyond
- **Charmonium**: Fine- and Hyperfine-Structure
- **Induced Interaction** and Effective Field Theory
- **String Breaking**:
  - ➡ Guidance from Lattice QCD
  - ➡ A schematic model

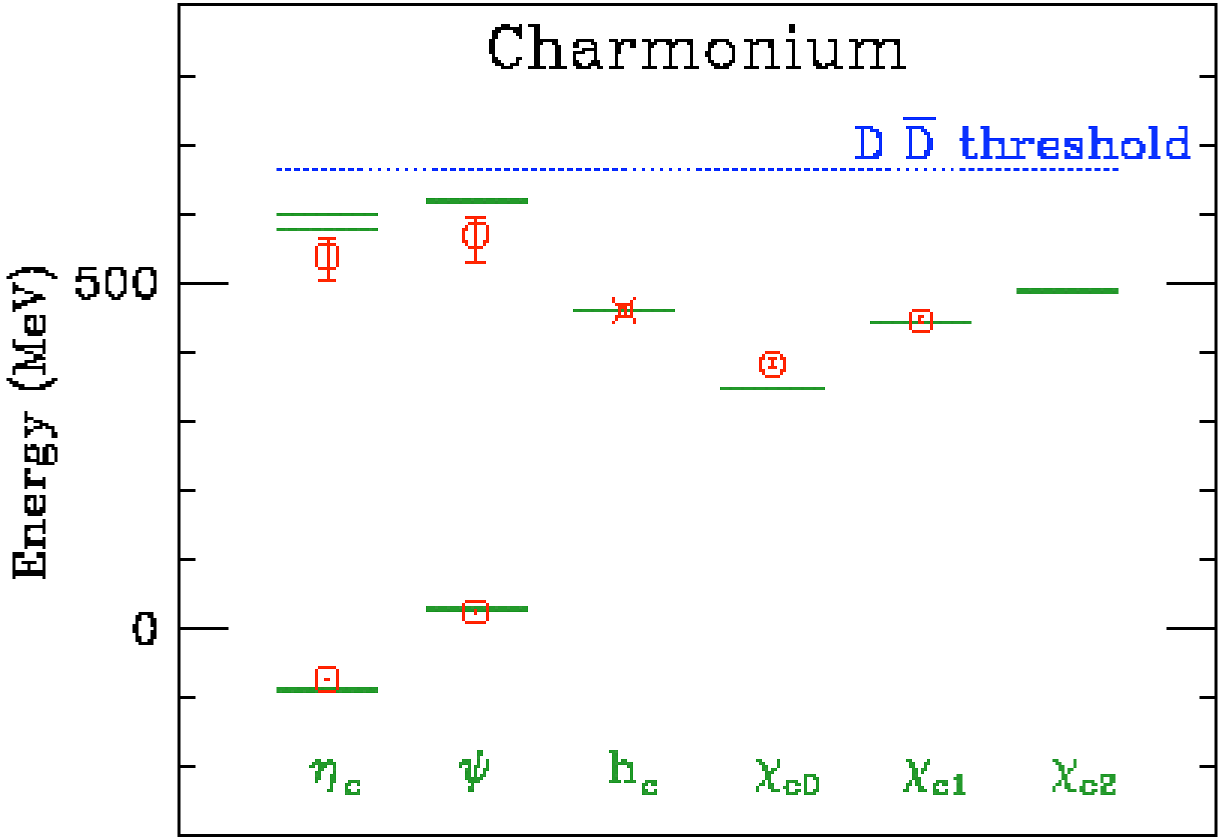
# Charmonium Spectroscopy




- “Is there anything NEW to be learned ?”
- Physics close to and in the $D\bar{D}$ continuum

# Lattice QCD

- Charmonium spectrum below $D\bar{D}$ threshold from lattice QCD with 3 light sea quarks, MILC configurations, improved (Fermilab) action



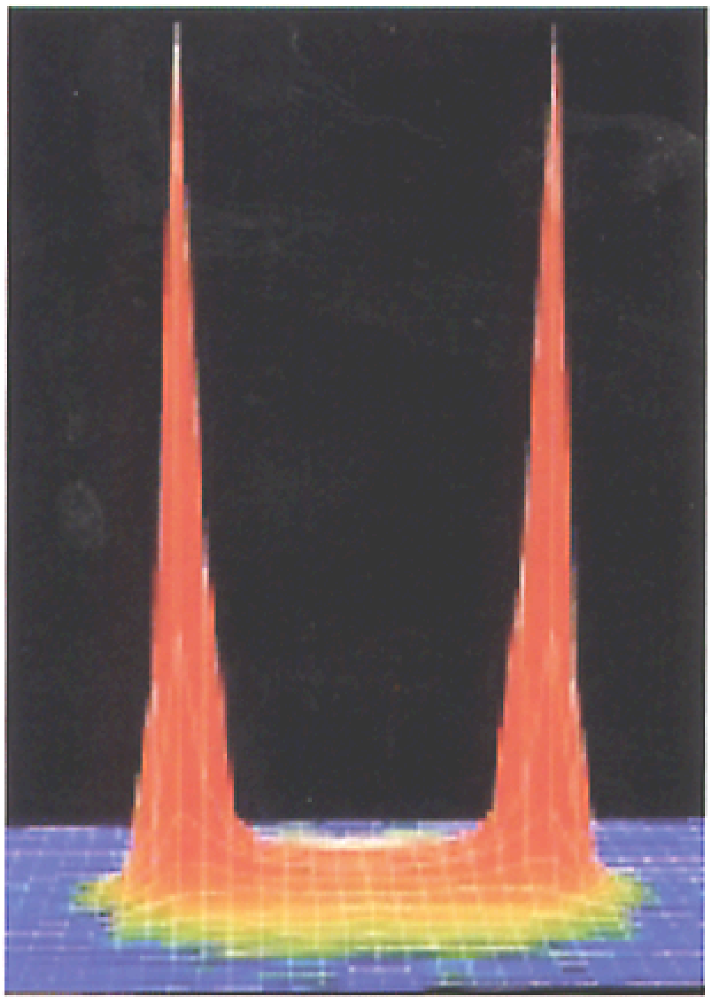

M. di Pierro et al., hep-lat/0310042

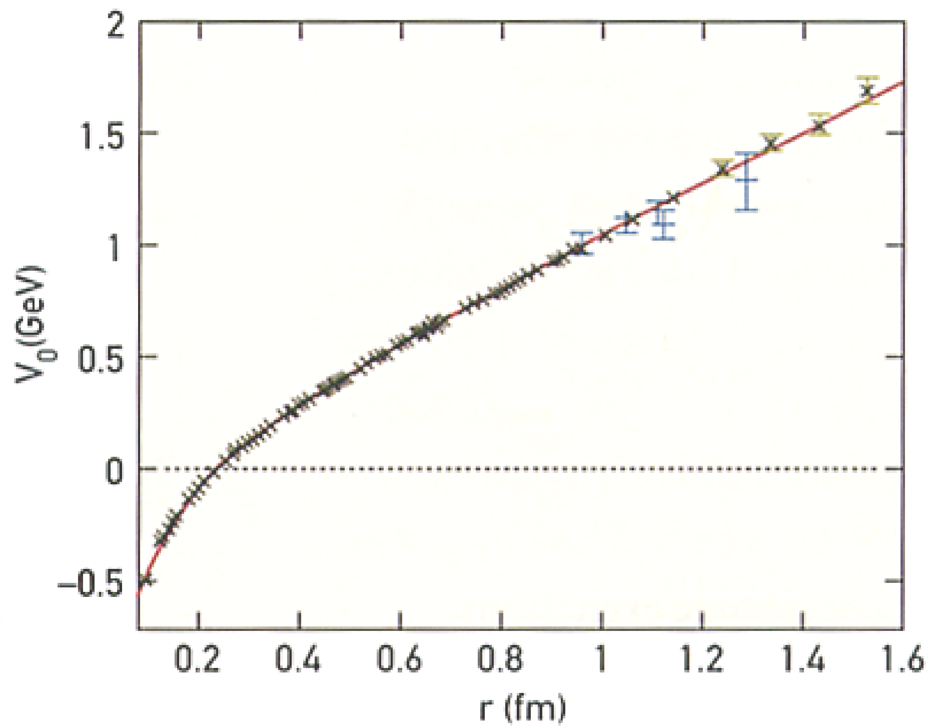
# Lattice QCD

Gluonic Flux Tube and Confining Potential
between heavy quark and antiquark

$$\mathbf{V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma\, r}$$

STRING TENSION $\sigma \simeq 1\,\mathrm{GeV/fm}$






G.S. Bali, Phys. Reports 343 (2001)1

1.

# MODELLING CHARMONIUM

- **Confinement** plus **Gluon Exchange** to **order** $\alpha_s^2$

- Non-perturbative **Induced** t-Channel **Interaction**

# Strategies

- Modern approach: **Effective Field Theory**

**Non-Relativistic QCD**:
Separation of scales provided by mass of heavy quark
Expansion in **v** (velocity) and $\alpha_s$

- **Potential** approach:

Bethe-Salpeter equation → non-relativistic reduction to order $m_c^{-2}$
→ Schrödinger equation:

$$\left[-\frac{\vec{\nabla}^2}{m_c} + U\right]\psi = (2m_c + E)\,\psi$$

$$U = \sigma\, r - \frac{4}{3}\,\frac{\alpha_s}{r}$$

$$+ V_{spin}\,\vec{s}_1 \cdot \vec{s}_2 + V_{tensor}\,\left(3\,\vec{s}_1 \cdot \hat{r}\;\vec{s}_2 \cdot \hat{r} - \vec{s}_1 \cdot \vec{s}_2\right) + V_{so}\,\vec{L} \cdot \vec{S} + \ldots$$

# Potential Models

- Early potential models of Charmonium based on

**Confinement + One-Gluon Exchange**

(Cornell, Richardson et al., Buchmüller et al. potentials and variants thereof)

used: $\sigma \simeq 1\,\mathrm{GeV/fm}$ $\alpha_s \simeq 0.4$

... together with LARGE c-quark mass:

$$m_c \simeq 1.5 - 1.8\,\mathrm{GeV}$$

- whereas:

$$m_c(\mu = m_c) = (1224 \pm 17 \pm 54)\,\mathrm{MeV}$$

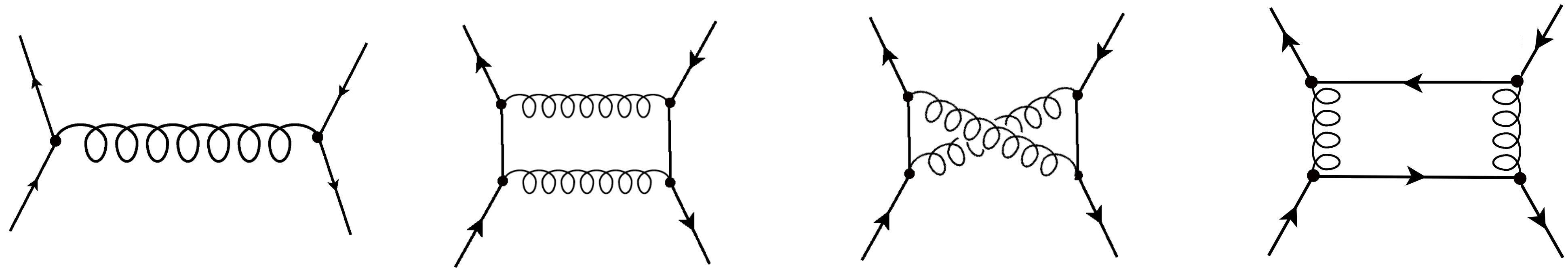
from inclusive semileptonic B decays

(Hoang and Manohar, PLB 633 (2006) 526)

- **Problems** with **spin-spin** & **spin-orbit** splittings

# Potential to order $\alpha_s^2$

(S.N. Gupta, S.F. Radford, W.W. Repko, Phys. Rev. D26 (1982) 3305)



$$\mathbf{U} = \mathbf{V}_{\mathbf{conf}} +$$

$$-\frac{4\alpha_s}{3r}\left[1-\frac{3\alpha_s}{2\pi}+\frac{\alpha_s}{6\pi}(33-2n_f)[\ln(\mu r)+\gamma_E]\right]+\frac{4\alpha_s}{3m^2 r}\left[1-\frac{3\alpha_s}{2\pi}+\frac{\alpha_s}{6\pi}(33-2n_f)[\ln(\mu r)+\gamma_E]\right]\nabla^2$$

$$+\frac{8\pi\alpha_s}{3m^2}\left[\left(1-\frac{3\alpha_s}{2\pi}\right)\delta(\vec{r})-\frac{\alpha_s}{24\pi^2}(33-2n_f)\nabla^2\left(\frac{\ln(\mu r)+\gamma_E}{r}\right)\right]-\frac{14\alpha_s^2}{9mr^2}$$

$$+\frac{32\pi\alpha_s}{9m^2}\vec{s}_1\cdot\vec{s}_2\left[\left(1-\frac{\alpha_s}{12\pi}(26+9\ln 2)\right)\delta(\vec{r})-\frac{\alpha_s}{24\pi^2}(33-2n_f)\nabla^2\left(\frac{\ln(\mu r)+\gamma_E}{r}\right)+\frac{21\alpha_s}{16\pi^2}\nabla^2\left(\frac{\ln(mr)+\gamma_E}{r}\right)\right]$$

$$+\frac{4\alpha_s}{m^2}\frac{\vec{s}_1\cdot\hat{r}\,\vec{s}_2\cdot\hat{r}-\frac{1}{3}\vec{s}_1\cdot\vec{s}_2}{r^3}\left[1+\frac{4\alpha_s}{3\pi}+\frac{\alpha_s}{6\pi}(33-2n_f)[\ln(\mu r)+\gamma_E-\tfrac{4}{3}]-\frac{3\alpha_s}{\pi}[\ln(mr)+\gamma_E-\tfrac{4}{3}]\right]$$

$$+\frac{2\alpha_s}{m^2}\frac{\vec{L}\cdot\vec{S}}{r^3}\left[1-\frac{11\alpha_s}{18\pi}+\frac{\alpha_s}{6\pi}(33-2n_f)[\ln(\mu r)+\gamma_E-1]-\frac{2\alpha_s}{\pi}[\ln(mr)+\gamma_E-1]\right]$$

# Potential to order $\alpha_s^2$ Results

(S.N. Gupta, S.F. Radford, W.W. Repko, Phys. Rev. D26 (1982) 3305)

| State | Mass (GeV) | exp. |
|---|---|---|
| $1\,^3S_1(\psi)$ | 3.097 | 3.097 |
| $1\,^1S_0(\eta_c)$ | 2.981 | 2.980 |
| $2\,^3S_1(\psi')$ | 3.685 | 3.686 |
| $2\,^1S_0(\eta_c')$ | 3.600 | 3.637 |

| State | Mass (GeV) | exp. |
|---|---|---|
| $1\,^3P_2(\chi_2)$ | 3.561 | 3.556 |
| $1\,^3P_1(\chi_1)$ | 3.515 | 3.511 |
| $1\,^3P_0(\chi_0)$ | 3.416 | 3.415 |
| $1\,^1P_1$ | 3.531 | 3.526 |

- Excellent agreement found with:

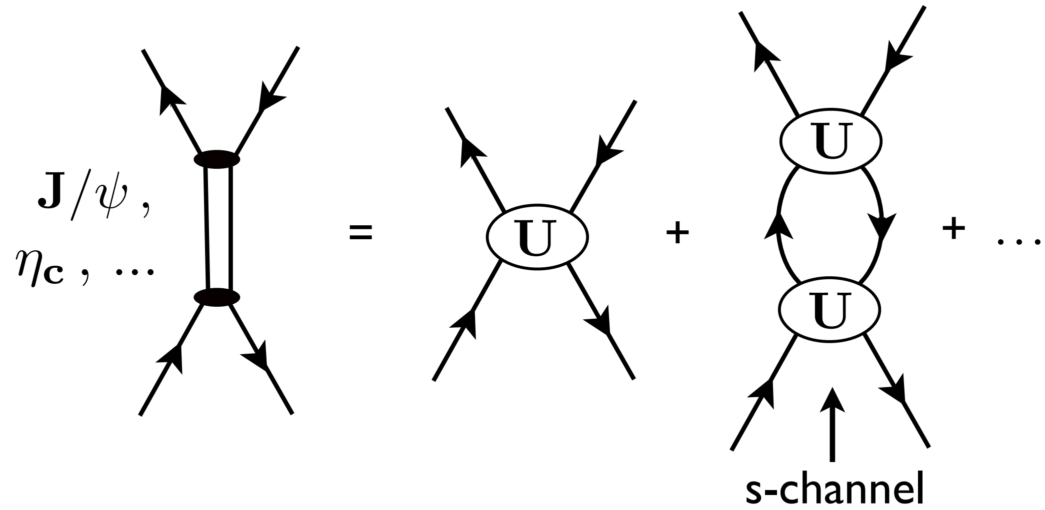
$$\sigma \simeq 0.9\,\mathrm{GeV/fm} \qquad m_c = 1.2\,\mathrm{GeV}$$

$$\alpha_s = 0.39 \quad \text{(large)}$$

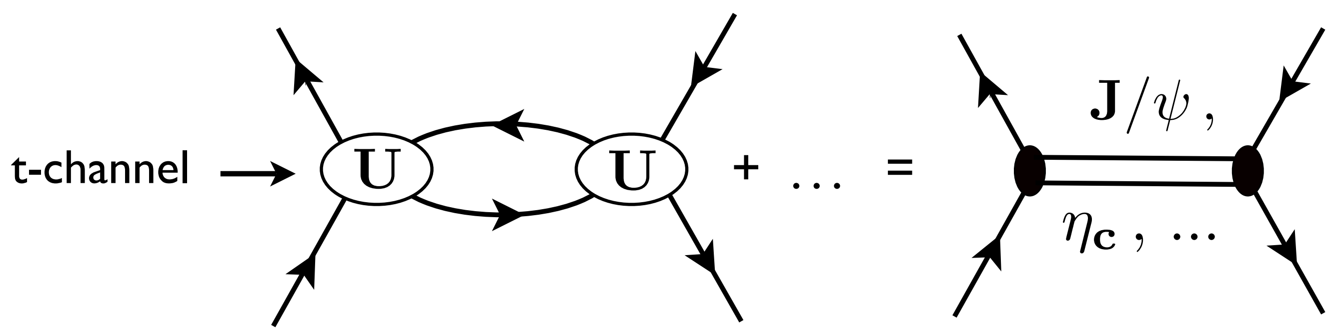
## Introducing the
# INDUCED INTERACTION

- Bethe-Salpeter equation:

  ... summing **LADDERS**:



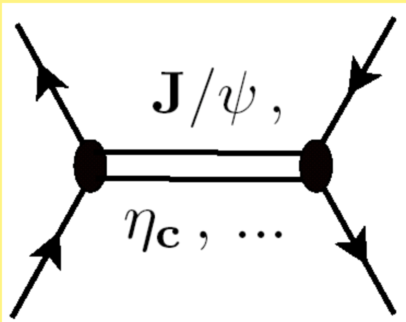

- Crossing: ... summing **BUBBLES**: induced non-perturbative interaction




- Effective Field Theory:

  absorbs induced interaction in **contact terms**

# INDUCED INTERACTION (contd.)




$$U_{ind} = \frac{G^2}{\vec{q}^2 + M^2}\left(1 - \frac{\vec{q}^2}{8m_c^2}\right) \text{(central)} \quad + \quad \text{spin-orbit}$$

$$- \frac{G^2}{4m_c^2\,(\vec{q}^2 + M^2)}\left[\vec{\sigma}_1 \cdot \vec{q}\;\vec{\sigma}_2 \cdot \vec{q} - (\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})\right]$$

$^1S_0$ (spin-longitudinal) $\qquad$ $^3S_1$ (spin-transverse)

- Coupling strength:

  related to asymptotic normalization $N_0$ of bound-state wave function

$$\alpha_{ind} \equiv \frac{G^2}{4\pi} = \frac{M^2}{m_c^3} N_0^2 \sim \mathcal{O}(1)$$

- Spin-dependent interaction:

$$U_{ind}(\vec{r}) = \frac{G^2}{48\pi\, m_c^2}\left(-M^2\,\frac{e^{-Mr}}{r} + 4\pi\,\delta^3(\vec{r})\right)\vec{\sigma}_1 \cdot \vec{\sigma}_1 \qquad \textbf{spin-spin}$$

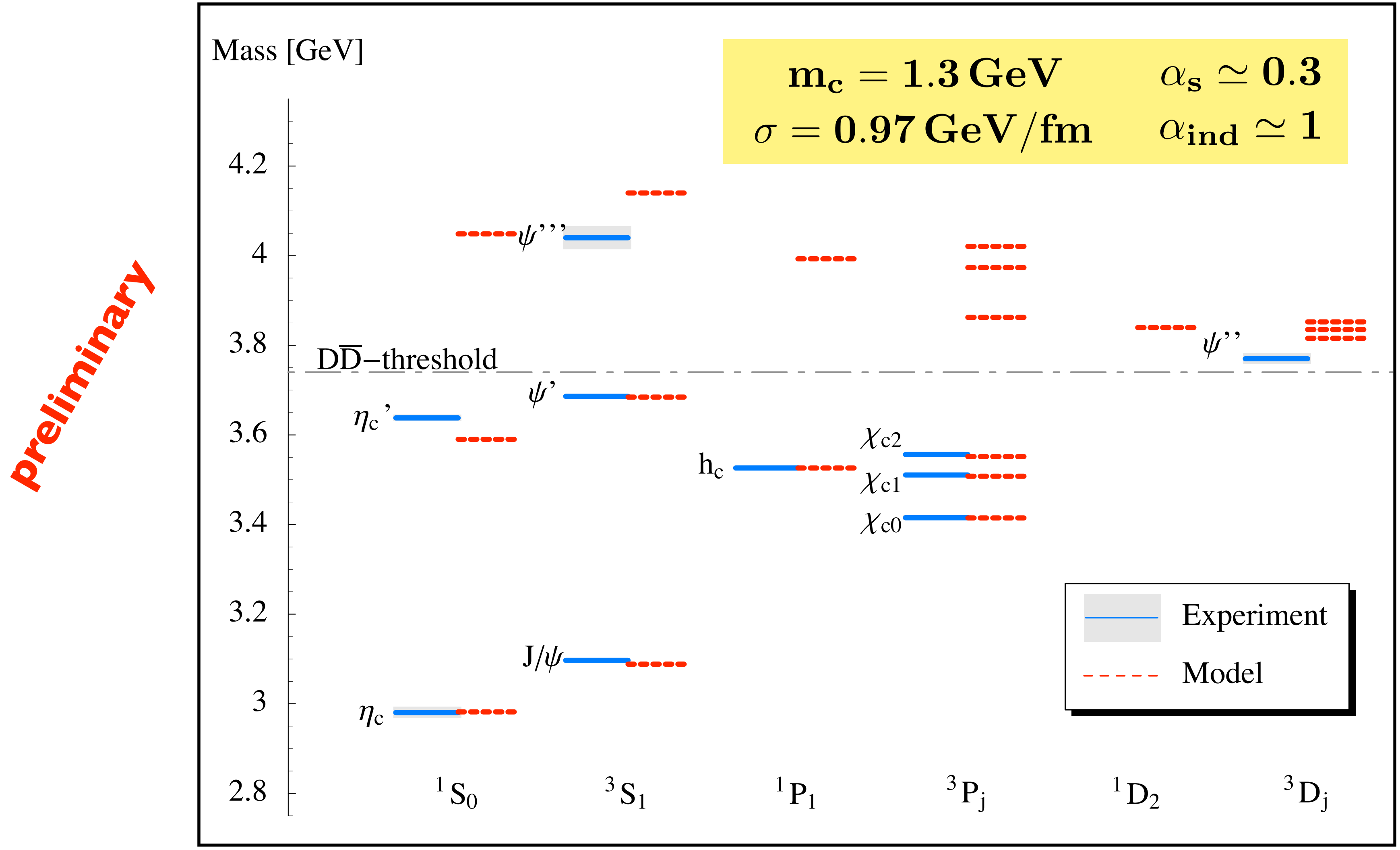
$$+ \frac{G^2}{24\pi}\,\frac{M^2}{m_c^2}\left(1 + \frac{3}{Mr} + \frac{3}{M^2 r^2}\right)\frac{e^{-Mr}}{r}\,S_{12}(\hat{r}) \qquad \textbf{tensor}$$

$$+ \frac{3\,G^2}{8\pi\, m_c^2}\left(M + \frac{1}{r}\right)\frac{e^{-Mr}}{r^2}\,\vec{L} \cdot \vec{S} \qquad \textbf{spin-orbit}$$

# Charmonium Spectroscopy with INDUCED interaction

(J. Eiglsperger, N. Kaiser, W.W.)

- confining potential + one- & two-gluon exchange + induced interaction



II.

# QUANTUM MECHANICS
# of
# STRING BREAKING

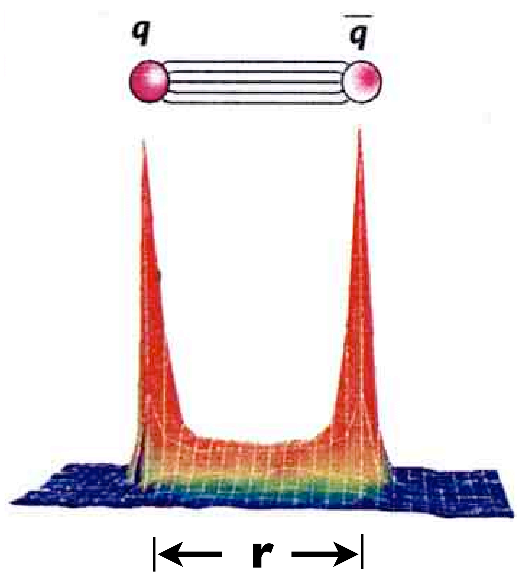
- Guidance from Lattice QCD
- Schematic Two-State Scenario
- Outlooks

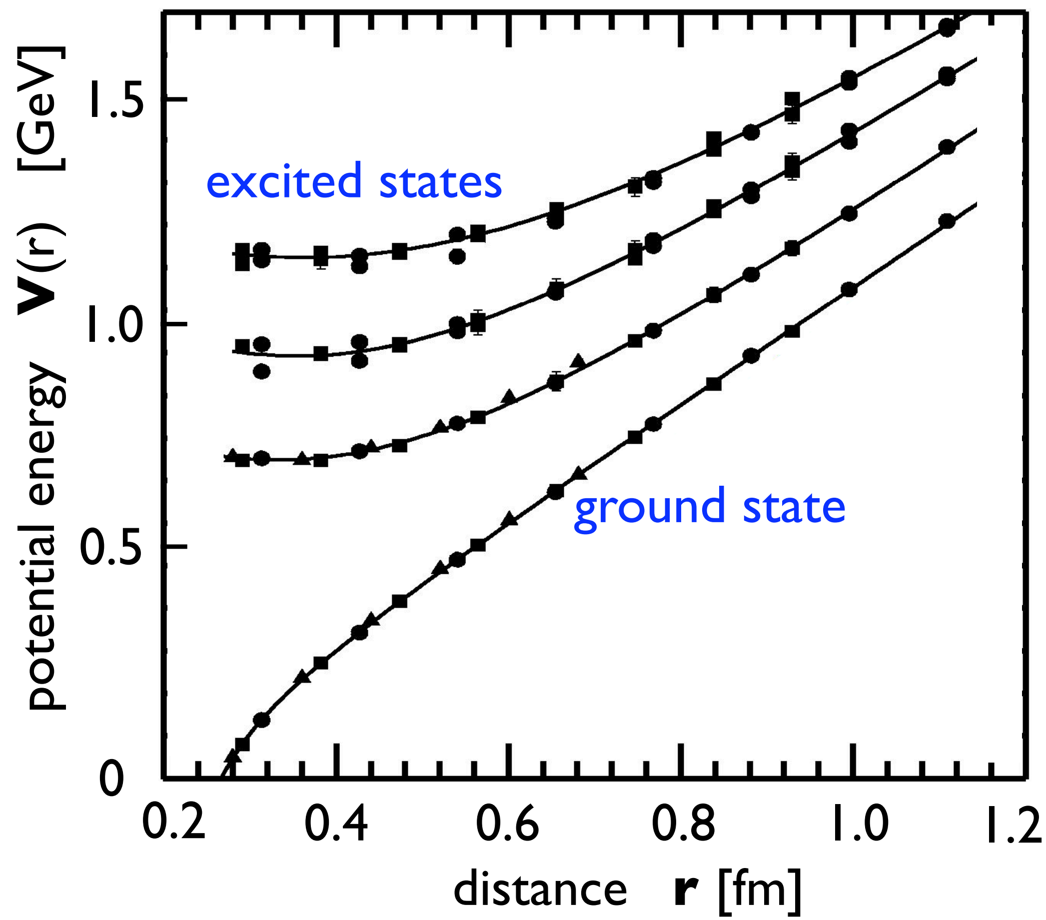
# CONFINEMENT

... the classic (but **incomplete**) picture:

**LATTICE - QCD:**
POTENTIAL between
(infinitely)
Heavy Quarks

(Action) Density of Color Fields



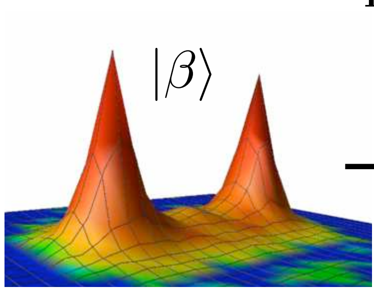



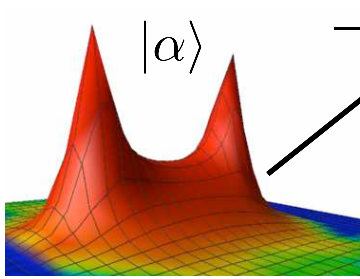

... at **r > 1 fm** the **STRING BREAKS**

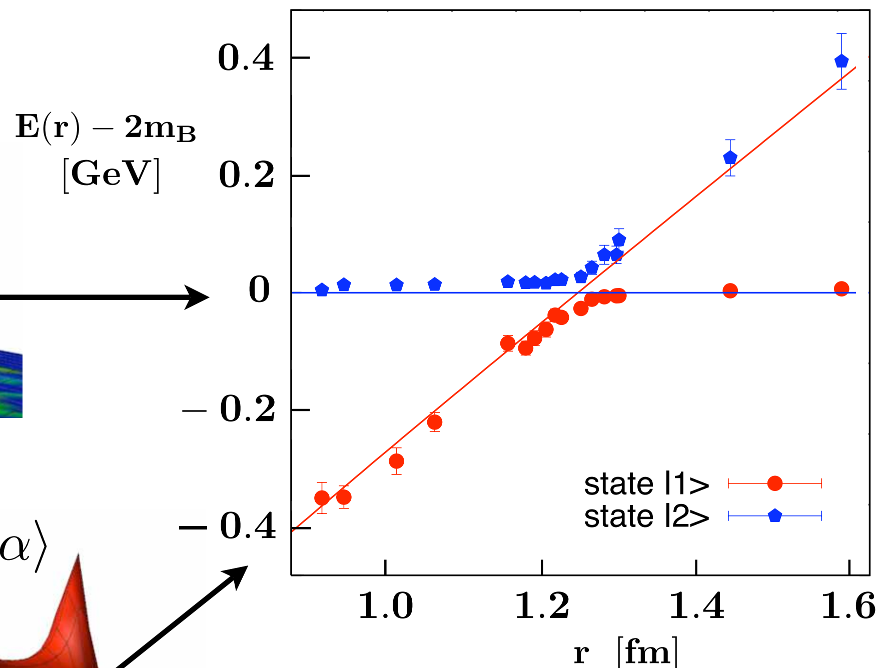
# STRING BREAKING in QCD

## Lattice QCD Results

(G. Bali et al.: Phys. Rev. D 71 (2005) 114513)




**schematic two-state scenario**

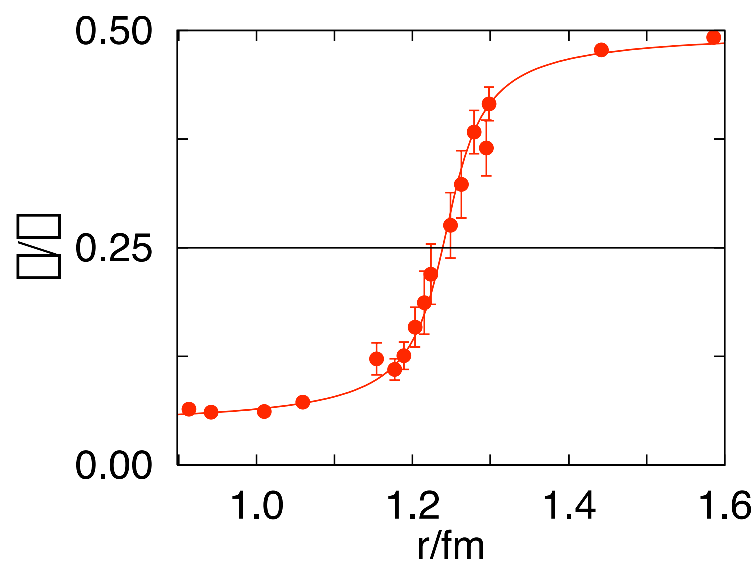
$$|\alpha\rangle = |\mathbf{Q\bar{Q}}\rangle \quad (\mathbf{e.g.}\ |\mathbf{c\bar{c}}\rangle)$$

$$|\beta\rangle = |[\mathbf{Q\bar{q}}][\mathbf{\bar{Q}q}]\rangle \quad (\mathbf{e.g.}\ |\mathbf{D\bar{D}}\rangle)$$

**mixing:**

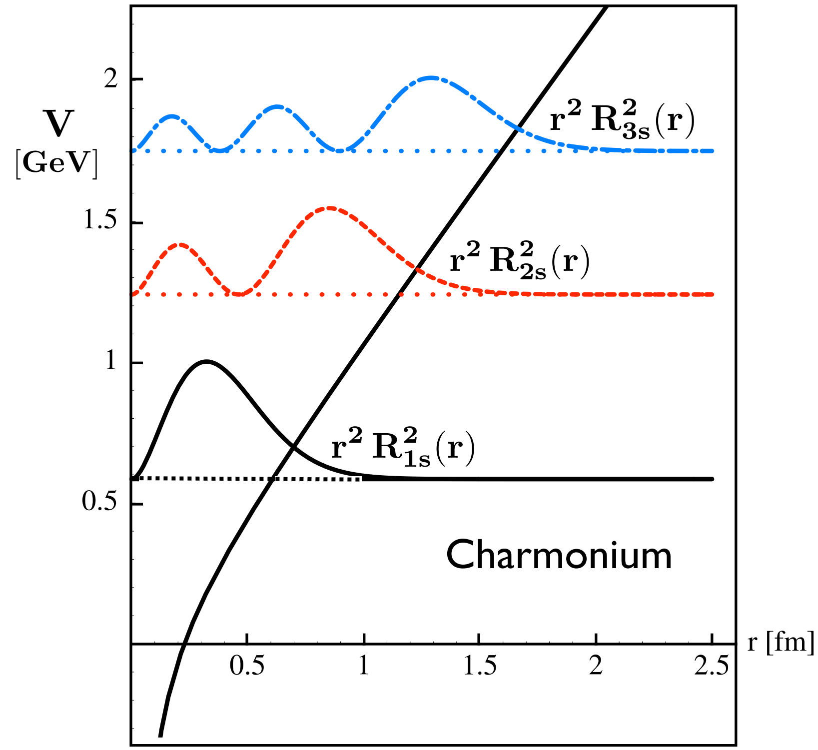
$$|\mathbf{1}\rangle = \ \ \mathbf{cos}\,\theta\,|\alpha\rangle + \mathbf{sin}\,\theta\,|\beta\rangle$$

$$|\mathbf{2}\rangle = -\mathbf{sin}\,\theta\,|\alpha\rangle + \mathbf{cos}\,\theta\,|\beta\rangle$$

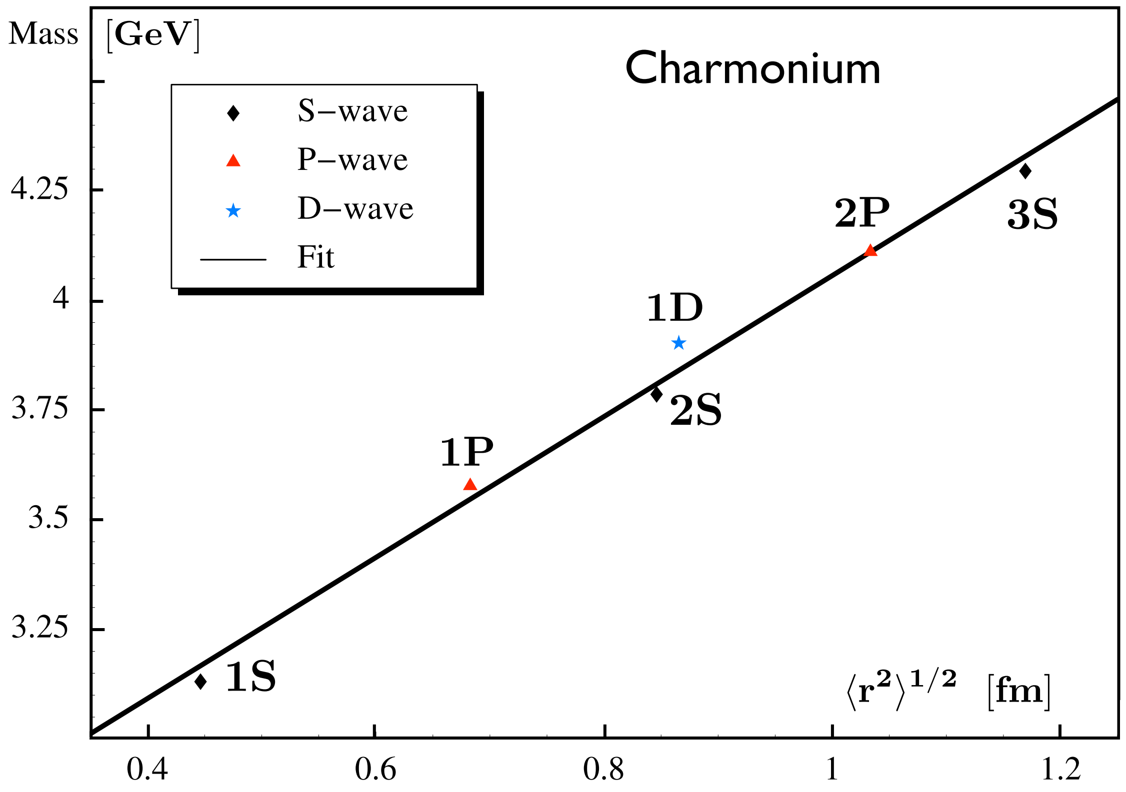


# MASS - RADIUS relations: Charmonium States

- Radial densities



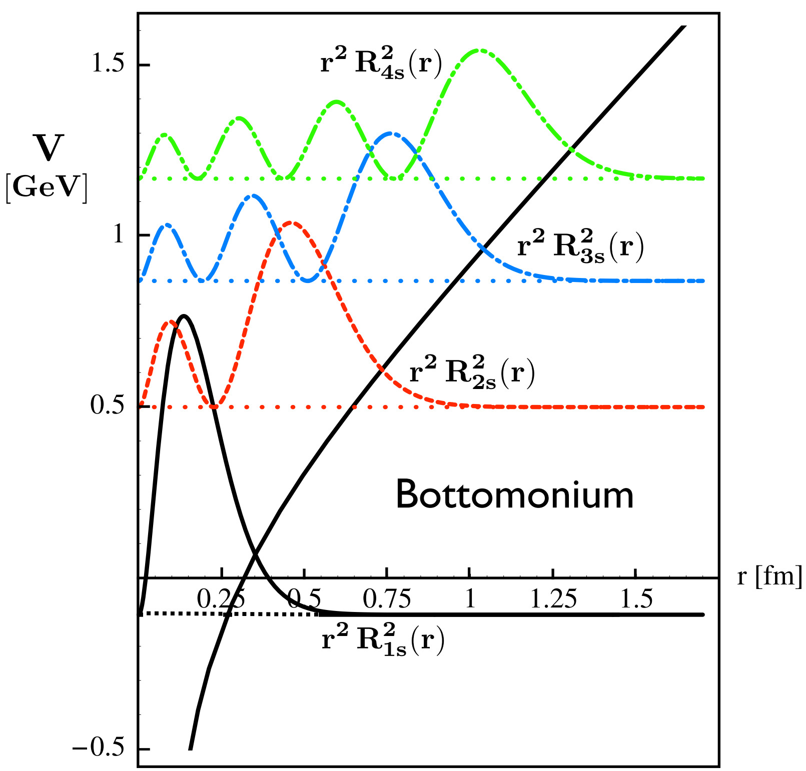

- Masses vs. root-mean-square radii



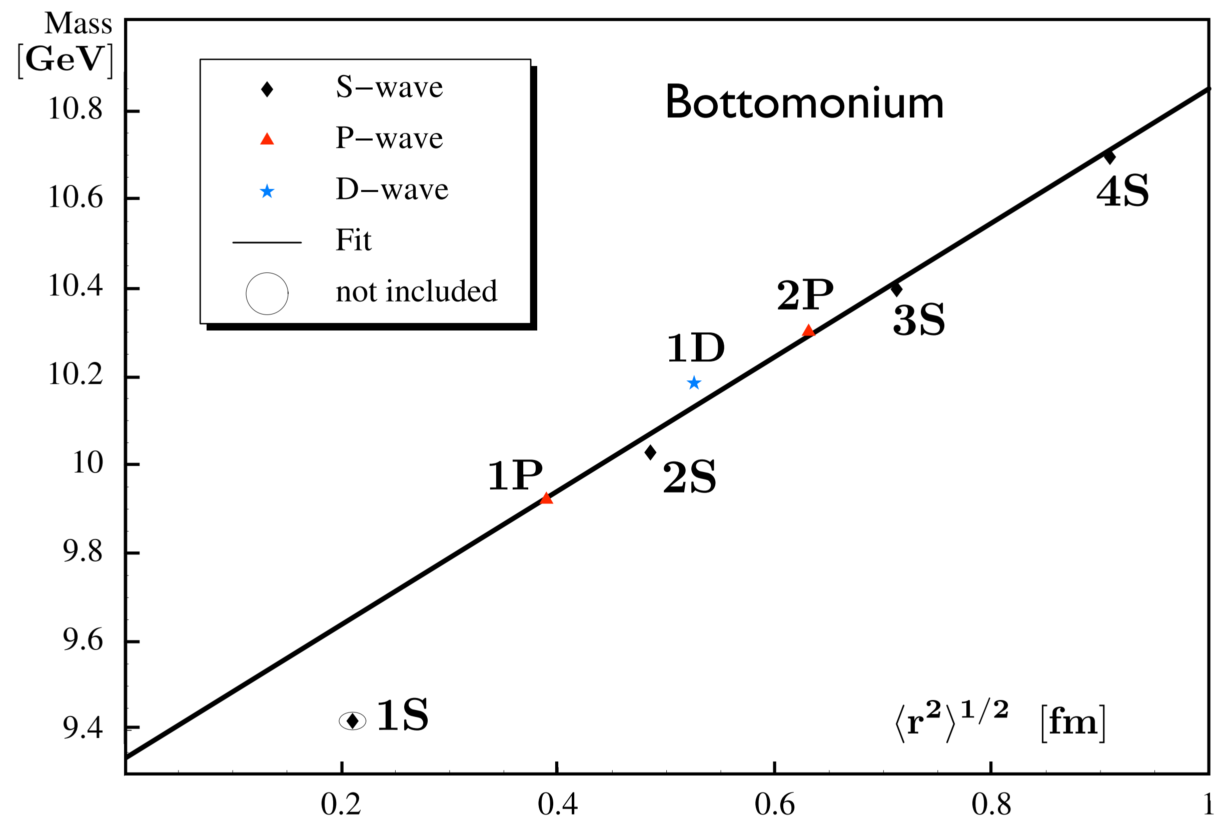

$$M = 2m_c + 1.67\,\text{GeV/fm} \cdot \langle r^2 \rangle^{1/2}$$

# MASS - RADIUS relations: Bottomonium States

- Radial densities

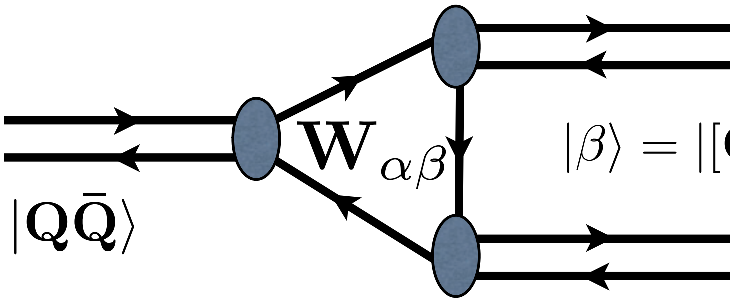



- Masses vs. root-mean-square radii




$$M = 2m_b + 1.5\,\mathrm{GeV/fm} \cdot \langle r^2 \rangle^{1/2}$$

# QM of String Breaking: Schematic Two-State Model




$|\alpha\rangle = |\mathbf{Q\bar{Q}}\rangle$

$|\beta\rangle = |[\mathbf{Q\bar{q}}][\mathbf{\bar{Q}q}]\rangle$

$$(\mathbf{H}_{\alpha\beta}) = \begin{pmatrix} \mathbf{2\,m_Q + c\,R} & \mathbf{W}_{\alpha\beta} \\ \mathbf{W}_{\beta\alpha} & \mathbf{2\,m_B} \end{pmatrix}$$

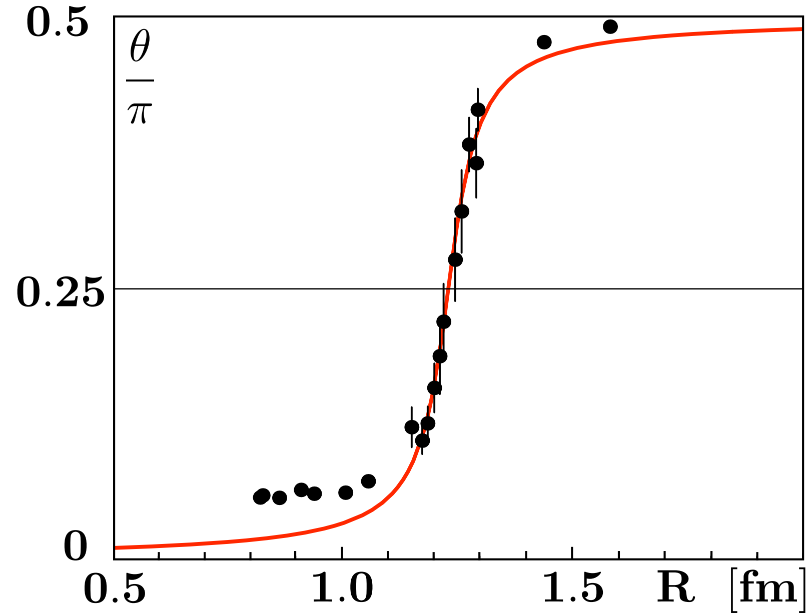
**mixing**:

$$|\mathbf{1}\rangle = \quad \mathbf{cos}\,\theta\,|\alpha\rangle + \mathbf{sin}\,\theta\,|\beta\rangle$$

$$|\mathbf{2}\rangle = -\mathbf{sin}\,\theta\,|\alpha\rangle + \mathbf{cos}\,\theta\,|\beta\rangle$$

$$\mathbf{tan}\,\mathbf{2}\theta = \frac{\mathbf{2W}_{\alpha\beta}}{\mathbf{H}_{\alpha\alpha} - \mathbf{H}_{\beta\beta}}$$

$$\mathbf{E_{1,2}} = \frac{\mathbf{1}}{\mathbf{2}}\left[\mathbf{H}_{\alpha\alpha} + \mathbf{H}_{\beta\beta} \pm \sqrt{(\mathbf{H}_{\alpha\alpha} - \mathbf{H}_{\beta\beta})^{\mathbf{2}} + \mathbf{4}|\mathbf{W}_{\alpha\beta}|^{\mathbf{2}}}\right]$$

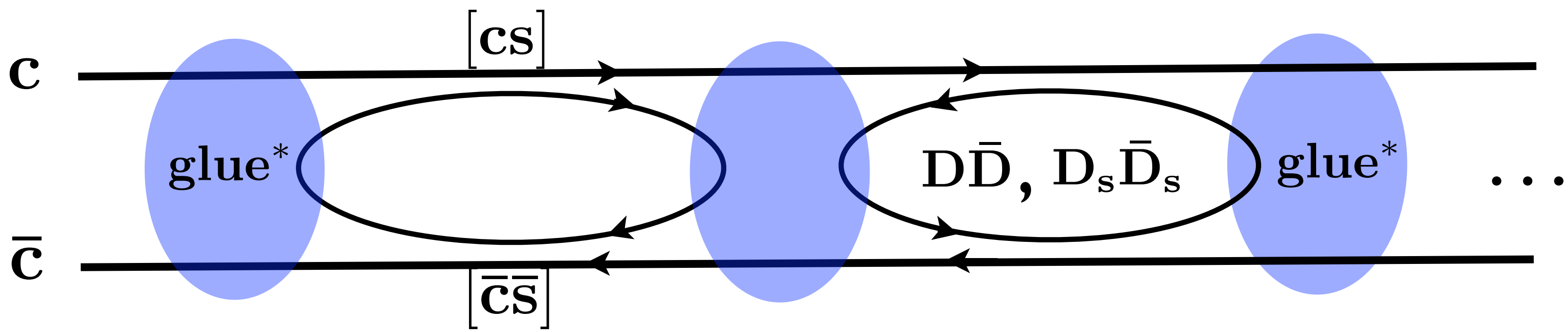



- example **bottomonium**:

**mixing / string breaking** matrix element $|\mathbf{W}_{\alpha\beta}| \simeq \mathbf{30\,MeV}$

# OUTLOOKS

- X, Y, Z states are likely to be **mixed** configurations of **four-quark**, **hybrid**, ... states:




$$|\mathbf{X}\rangle = \mathbf{a_1}|\mathbf{c\bar{c}}\rangle + \mathbf{a_2}|[\mathbf{c\bar{q}}][\mathbf{\bar{c}q}]\rangle + \mathbf{a_3}|[\mathbf{cq}][\mathbf{\bar{c}\bar{q}}]\rangle + \mathbf{a_4}|\mathbf{c\bar{c}g^*}\rangle + \ldots$$

- **Experiment**: high-precision measurements of decays

- **Theory**: coupled - channels approach combined with Lattice QCD and effective field theory methods

- **Charmonium** states above threshold: **complex** potential

$$\mathbf{U_{eff}} = \mathbf{U_0} + \mathbf{W}^\dagger \frac{\mathbf{1}}{\mathbf{E} - \mathbf{H_0} + \mathbf{i}\varepsilon} \mathbf{W}$$