Helicity Amplitudes for Final State Particle with non-zero Spin

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Helicity Formalism

1) Two body decay $(P,J,M) \rightarrow (p_1,s_1,\lambda_1) \otimes (p_2,s_2,\lambda_2)$

$$A_{M;\,\lambda 1\,\lambda 2}(\theta,\phi) = D^{J}_{M\,\lambda}{}^{*}(\theta,\phi)\; A_{\lambda;\,\lambda 1\,\lambda 2}(0,0)$$

(Eq. 1)

$\lambda = \lambda_1 - \lambda_2$

$A_{\lambda;\,\lambda 1\,\lambda 2}(0,0) = \langle J\lambda;\, LS\, \mathbf{0}\lambda\rangle$

$\langle J\lambda;\, s_1 s_2 \lambda_1 - \lambda_2\rangle\; BW_L$

$D^{J}_{M\,\lambda}(\theta,\phi) = D^{J}_{M\,\lambda}(\phi,\theta,0)$

((J+1)-dimensional representation of the frame rotation, see QM)

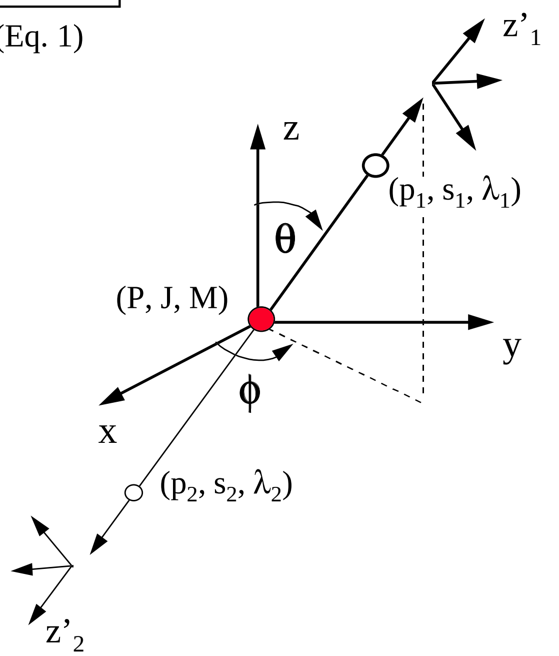


Fig 1

2) Decay Chain, Ex: $p\bar{p} \rightarrow b_1\,\pi,\; b_1 \rightarrow \omega\pi,\;\; \omega \rightarrow \pi\,\gamma$

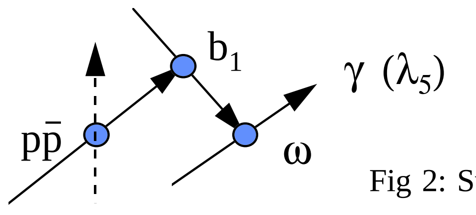


Fig 2: Subsequent frame transformation

$$A = A_{M;\,\lambda 5\,0} = A^{JLS}_{M;\,\lambda 1\,0}(p\bar{p}) * A^{jls}_{\lambda 1;\,\lambda 3\,0}(b_1) * A^{111}_{\lambda 3;\,\lambda 5\,0}(\omega) \quad \text{(Eq. 2)}$$

3) Transition Probability

$$w_D = \mathrm{Tr}\,\rho_f = \mathrm{Tr}\,(T\,\rho_0\,T^{+})\,; \quad T = \alpha_i A^i$$

(Eq. 3)

Problem of Calculating the Transition Probability

1) Ex: $pp(1^{--}) \rightarrow b_1 \pi$ (L=0); $b_1 \rightarrow \omega\pi$ (l=0); $\omega \rightarrow \pi\gamma$

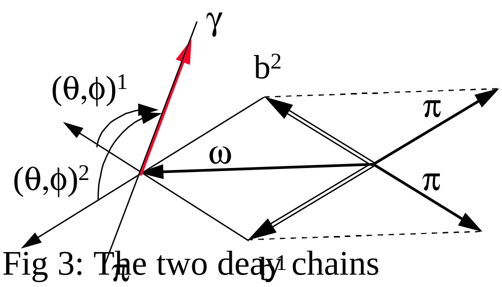


Fig 3: The two decay chains

2 combinatorial possibilities:

b^1: $A^1_{M;\lambda 0}$,

b^2: $A^2_{M;\lambda 0}$

$$T_{M;\lambda 0} = A^1_{M;\lambda 0} + A^2_{M;\lambda 0}$$

2) MC Simulation vs. QM

$$w_{MC} = w_{PS} \times \varepsilon \times w_D$$

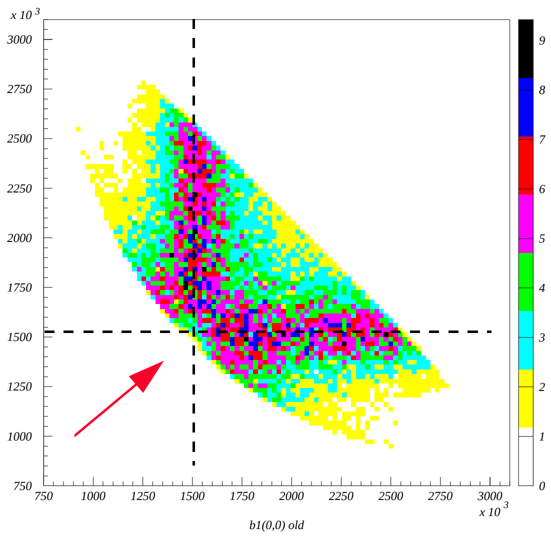
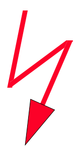


Fig 4: Simulated Dalitz plot

MC: There is a destructive behavior at the crossing of the two b_1 bands



QM: Because of ***L*=0** and ***l*=0** one would expect **no angular dependence** (interference only determined by the dynamics, eg BW)*

* **J** = $\mathbf{S}_\omega$; $1^{--} \rightarrow 1^{+-}\, 0^{-+}$, $1^{+-} \rightarrow 1^{--}\, 0^{-+}$, $1^{--} \rightarrow 1^{--}\, 0^{-+}$

The Correct Transition Amplitude

The frame (assumed in eq.2) of the final state (ω) depends on the decay chain!
The difference between the chains is a rotation around the z axis which translates into an artificial phase (see fig 5).

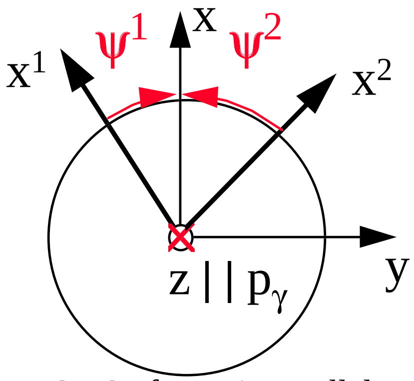


Fig 5: CMS of ω**,** z is parallel to the γ direction ω (see also fig 2)

Frames:

Σ^1: (x^1, y^1, z)

Σ^2: (x^2, y^2, z)

Σ: (x,y,z) reference system

Solution: For each event and each chain the angle ψ has to be calculated and the transition amplitude has to be corrected by the phase Exp(i λ_f $\omega\psi^j$) (see Eq. 4).

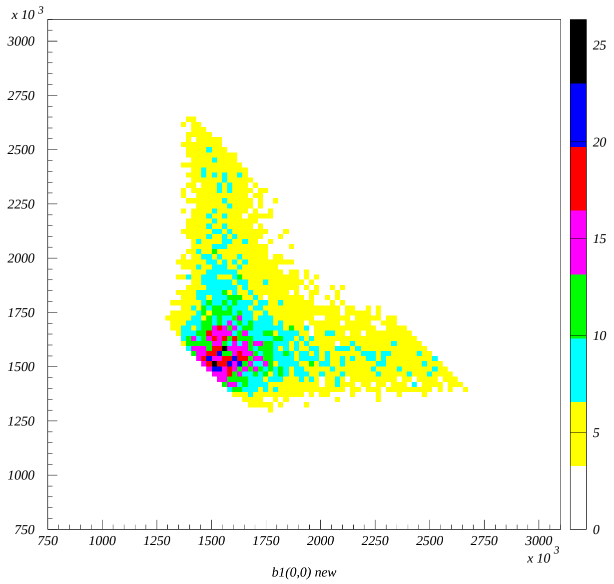


Fig 6: Simulated Dalitz plot (corrected Transition)

$$A^j \rightarrow A^j \,\mathrm{Exp}(\, i\, \lambda_f\, \psi^j) \quad \text{(Eq. 4)}$$

Conclusion

- No correction is needed for **scalar** final state (λ=0 !)
- For final states **with spin** the alignment of the final state frame has to be considered rigorously , the later can change the phase of the transition amplitude.
  In order to compensate a rotation around z, the phase correction $exp(i \lambda_f \psi)$ has to be applied (see eq. 4) .
- Nevertheless the helicity formalism is correct, if correctly applied!

Further Reading (Selection)

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